MATH-328 Methods of Applied Mathematics

Logistic Model

DO NOT USE SOFTWARE IN PARTS 1, 2, 3.

The **logistic equation** models the population dynamics of a single species subject to intra–specific competition:

$$\frac{dp}{dt} \;=\; rp\Big(1 - \frac{p}{K}\Big)\,, \tag{1}$$

subject to the initial population

$$p(0) \;=\; p_0\,. \tag{2}$$

1. The ODE (1) can be solved using two techniques you learned in Differential Equations (Math-204):

   - by separable variables, and
   - as a Bernoulli equation.

   Use either technique to solve this to obtain the solution $p(t)$ explicitly in terms of t. After applying the initial condition (2), show that the solution can be expressed as

   $$p(t) \;=\; \frac{K}{1 + (K/p_0 - 1)e^{-rt}}\,. \tag{3}$$

2. By hand, verify that $p(0) = p_0$.

3. By hand, use the solution (3) to determine

   $$\lim_{t\to\infty} p(t)\,.$$

   In a complete and coherent sentence, explain what this result means.

4. If $r = 0.05$, $K = 10000$, and $p_0 = 20$, use software (Maple?, Matlab?) to plot $p(t)$ on the interval $0 \le t \le 300$.