Sec. 4.3

1.a) $J(y) = \int_0^1 y' dx$, $y(0)=0$, $y(1)=1$.

The Lagrangian is,

$$L = y',$$

so $L_y = 0$, $L_{y'} = 1$.

E-L eqn:

$$L_y - \frac{d}{dx}(L_{y'}) = 0$$

$$\Rightarrow \quad 0 - \frac{d}{dx}(1) = 0$$

$$\Rightarrow \quad 0 = 0.$$

__ALL__ ftns $y \in C^2[0,1]$ satisfy the E-L eqn.

So extremals are the set of all twice continuously differentiable ftns that satisfy the two BC's: $y(0)=0$, $y(1)=1$.

1b) $J(y) = \int_0^1 yy' dx$, $y(0)=0$, $y(1)=1$.

$$L = yy'$$

$$\Rightarrow \quad L_y = y', \quad L_{y'} = y.$$

E-L eqn:

$$L_y - \frac{d}{dx}(L_{y'}) = 0$$

Gives $y' - \frac{d}{dx}(y) = 0$

$$\Rightarrow \quad y' - y' = 0$$

$$\Rightarrow \quad 0 = 0.$$

So as in (1a), extremals are ALL twice ~~continous~~ continuously differentiable ftns that satisfy the BC's $y(0)=0$, $y(1)=1$.