To approximate the solution of the 1st order IVP:

$$y' = f(x,y), \qquad y(x_0) = y_0 .$$

Suppose we have approximated y_n at node x_n, and now we seek to approximate y_{n+1} at node x_{n+1}.

Calculate these in the order given:

$$
\begin{aligned}
& x_n = x_0 + nh, \qquad\qquad x_{n+1} = x_0 + (n+1)h, \\
k_1 &= h\, f(x_n,\, y_n), \\
& r = x_n + \frac{1}{4}h, \qquad s = y_n + \frac{1}{4}k_1, \\
k_2 &= h\, f(r,\, s), \\
& r = x_n + \frac{3}{8}h, \qquad s = y_n + \frac{3}{32}k_1 + \frac{9}{32}k_2, \\
k_3 &= h\, f(r,\, s), \\
& r = x_n + \frac{12}{13}h, \qquad s = y_n + \frac{1932}{2197}k_1 - \frac{7200}{2197}k_2 + \frac{7296}{2197}k_3, \\
k_4 &= h\, f(r,\, s), \\
& r = x_n + h, \qquad\qquad s = y_n + \frac{439}{216}k_1 - 8\,k_2 + \frac{3680}{513}k_3 - \frac{845}{4104}k_4, \\
k_5 &= h\, f(r,\, s), \\
& r = x_n + \frac{1}{2}h, \qquad s = y_n - \frac{8}{27}k_1 + 2\,k_2 - \frac{3544}{2565}k_3 + \frac{1859}{4104}k_4 - \frac{11}{40}k_5, \\
k_6 &= h\, f(r,\, s), \\
\widehat{y}_{n+1} &= y_n + \left(\frac{25}{216}k_1 + \frac{1408}{2565}k_3 + \frac{2197}{4104}k_4 - \frac{1}{5}k_5 \right), \qquad \text{Error} = \mathcal{O}(h^4) \\
y_{n+1} &= y_n + \left(\frac{16}{135}k_1 + \frac{6656}{12825}k_3 + \frac{28561}{56430}k_4 - \frac{9}{50}k_5 + \frac{2}{55}k_6 \right), \qquad \text{Error} = \mathcal{O}(h^5) \\
|\text{Error}| &\approx |y_{n+1} - \widehat{y}_{n+1}|, \qquad \text{(gives an error estimate for } y \text{ at node } x_{n+1}\text{).}
\end{aligned}
$$

Notes:

1. We keep the value y_{n+1} at node x_{n+1}.

2. "**Error**" gives an **error estimate** for y_{n+1} at node x_{n+1}. Using x_{10} as an example: if the error estimate exceeds our tolerance at node x_{10} (indicates that h is too large), then we return to node x_9, reduce h, and recalculate y_{10} at the new node x_{10}.

   On the other hand, if the error estimate at node x_{10} is significantly smaller than our tolerance (indicates that h is much smaller than necessary), we may increase h and proceed to calculate y_{11} at node x_{11}. (In this case there's no need to back up to recalculate y_{10}.)