MATH–305 PROGRAM ASSIGNMENT 3 Fall 2011

Due Thursday, December 8

- **Review the document entitled, “Programming Projects” and ”Program Requirements.”**
- **Do not begin until you have the codes for the Euler method and the modified Euler method working and giving correct results. Also see the Maple examples and assignments on the course web site.**
- **Use formatted output. See the course web site.**

PART I: Testing a Sample Problem

Set Digits to 16 at the top of your Maple code.

Write a program that uses the **classical Runge–Kutta** method to solve an initial value problem

$$\frac{dy}{dx} = f(x,y)\,, \qquad y(x_0) = y_0\,. \tag{1}$$

The function $f(x,y)$ should be written as a Maple function, not as a Maple expression. (See my examples or the Maple Help Sheets.) The following quantities should be entered at the top of the code:

- function $f(x,y)$ for the DE,
- the interval endpoints a and b on which to solve the problem (a is x_0),
- the initial value y_0, and
- the number of steps N.

At each node the program should print

- the node number $(1,2,3,\ldots)$,
- the node x, and
- the approximate solution y at that node.

1. Use your program to solve the test problem:

$$\frac{dy}{dx} = \sqrt{x} + \cos y\,, \qquad y(2) = 3\,, \tag{2}$$

   on the interval $[2,6]$ using

   (a) $N = 4$ steps, and
   (b) $N = 8$ steps.

   Your results for this sample problem should agree with those posted on the course web site.

2. In Step (1b), plot the results for y versus x at each node using the point style.

PART II

Set Digits to 16 at the top of your Maple code. All numerical results should include proper and appropriate units.

In Project 2 we investigated the annual energy yield of a single wind turbine. Our model there assumed that the power depends only on the wind speed. However, the effectiveness of the wind to drive the rotor also depends on the air density. Less dense air moving at 7.1 m/s is less effective at driving the rotor than more dense air moving at the same speed. In Project 2 we used an air density of 1.225 kg/m^3, which is the average air density at sea level. Clearly, though, air density is lower at higher elevations. It is also lower at higher a dew point (higher relative humidity).

Here we will assume that our wind farm is at an elevation of 1500 m above sea level at a high dew point. The air pressure P varies with elevation z above sea level according to the model (IVP):

$$\begin{aligned} \frac{dP}{dz} &= -\frac{g\,P}{R\,(T_0 - k\,z)\,(1+\frac{z}{r})^2}\,, \qquad a \le z \le b\,, \\ P(0) &= P_0\,. \end{aligned} \tag{3}$$

This model accounts for the fact that air density, air temperature, air pressure, and even gravity are all functions of elevation z.

The values of the parameters are:

- $g = 9.81$ m/s^2 is the gravitational acceleration at sea level,
- $R = 287.053$ m^2/(K $\cdot$ s^2) is the specific gas constant for dry air,
- $r = 6{,}356{,}766$ m is the Earth's mean radius at sea level,
- $T_0 = 288.15$ K is the air temperature adjusted to sea level,
- $k = 0.00649$ K/m is the temperature lapse rate for dry air,
- $P_0 = 1013.25$ mb is the air density at sea level,
- z is the elevation (in meters) above sea level ($z = 0$ at sea level), and
- P is the air pressure [mb] at elevation z.

Use your **classical Runge–Kutta** code to solve for pressure P versus z on the interval $0 \le z \le 1500$ m using:

1. $N = 10$ steps, and
2. $N = 20$ steps.
3. At common nodes, how much more accurate do we expect the results in Step 2 to be than those from Step 1? For the $N = 20$ case,
   (a) plot the results for P versus z using the line style, and
   (b) plot the results for P versus z using the symbol style.
4. The atmospheric density ρ is a function of pressure P and dew point dp via

$$\rho = \frac{100}{R\,(T_0 - kz)}\left(P - 2.308928054 \times 10^{\frac{7.5\,dp}{237.3\,+\,dp}}\right)\,, \tag{4}$$

   where P must be in mbars and dew point dp must be in $^\circ$C.
5. Based on Step 2, what is the air density at the wind farm if the dew point is 25°C? State your answer in text mode.
6. Now input this density from Step 5 into the Simpson's code you wrote for Project 2. (Make sure you corrected any errors in that code first!) Assume again that the average annual wind speed is 7.1 m/s, the rotor diameter is 40 m, and the turbine efficiency is 28%. As before, use 72 subintervals to approximate the integral. (You need not use Richardson extrapolation.)
   (a) What is the annual energy yield for the wind turbine at this higher elevation and high dew point? Include proper units.
   (b) Compared to the correct result in Project 2, what is the decrease in annual energy yield at this higher elevation and higher dew point?

   You will turn in these results also, but you need not answer every question from Project 2.

Note that this engineering problem involves integration, differential equations, thermodynamics, chemistry, meteorology, physics, and two different topics from numerical methods. It is very common for a single problem to involve different scientific and mathematical fields. One could even say that this problem is more about meteorology than about engineering.