Due Tuesday, Nov. 30

- **Review the document entitled, "Programming Projects."**
- **Review the "Late Penalty Schedule" in the course policy.**
- **You should not attempt this until you have the code for the trapezoidal rule working and giving correct results. Also see the Maple examples and assignments on the course web site.**
- **All results should be expressed in decimal form.**

Here you will write a Maple program (see the examples on the course web site) for **Simpson's–1/3** rule to approximate the integral

$$\int_a^b f(x)\,dx\,. \tag{1}$$

The code should be flexible enough to allow the user to easily change these quantities at the top of the code:

- the function $f(x)$ to be integrated,
- the interval endpoints a and b, and
- the number of subintervals n dividing interval $[a, b]$.

PART I — Testing a Sample Problem

Set `Digits` to 20 at the top of your Maple code.

1. Use Maple's `int` command to exactly evaluate the integral

$$\int_{-2}^{4} (4\,x^3 - 9\,x^2 - 8\,x + 21)\,dx\,. \tag{2}$$

2. Now use your Simpson's–1/3 rule code to evaluate the integral (2) using $n = 12$ subintervals. Remember that Simpson's–1/3 rule should correctly integrate a cubic polynomial! So if your Maple code is written correctly, the result you obtain here must agree with the result that you obtained in Step 1 (except for truncation error in the last digit or two). **Do NOT proceed if this result does not agree with that obtained in Step 1.**

You will hand in all of these results.

PART II

When harvesting wind energy via turbines, the goal is to maximize the wind power. From fluid mechanics, we know that the power (units of W) is related to wind speed v, air density ρ, and swept area A (the disk shaped area swept by the rotor) via

$$P \;=\; \frac{1}{2}\,A\,\rho\,v^3 \;=\; 0.6125\,A\,v^3\,, \qquad \text{(we're using } \rho = 1.225 \text{ kg/m}^3 \text{ for air density)}\,.$$

Simply put, the power is proportional to the cube of wind speed: $P = \mathcal{O}(v^3)$. For example, if the wind speed is 5 m/s one day and 10 m/s the second day, then the wind power is $(10/5)^3 = 8$ times greater the second day. Even a small increase in wind speed can substantially increase the wind power. For example, if the wind speed is 6 m/s at Farm A and 7 m/s at Farm B, then $\frac{P_B}{P_A} = (\frac{7}{6})^3 \approx 1.6$. In this case, a mere 17% increase in wind speed produces a 60% increase in available wind power. Thus, it is extremely important to place wind farms at windier sites.

At a given location, obviously the wind speed is not constant but varies continuously. How do we account for this varying wind speed when determining available wind power? Specifically, does the wind speed represent one instant or an average over some time period? The use of average wind speed presents an added complication. To illustrate this, consider two scenarios:

- **Wind Farm A:** The wind speed is 10 m/s all day.
- **Wind Farm B:** The wind speed is 5 m/s for half the day and 15 m/s the other half.

Both farms experience an average wind speed of 10 m/s, so if we merely use the average wind speed to determine available wind power, we would obain $P \propto 10^3 = 1000$ in both cases. However, at Farm B the *true* wind power is $P_B \propto \frac{1}{2}(5^3 + 15^3) = 1750$. This simple example demonstrates that using average wind speed is not sufficient. Instead, we must account for the **distribution of wind speed over time**.

For temperate climates (like much of the continental U.S.), the wind speed closely follows a Rayleigh distribution:

$$f \;=\; \frac{\pi\, v}{2\, V^2}\, e^{-\frac{\pi}{4}\left(\frac{v}{V}\right)^2}\,, \tag{3}$$

where

- V is the average wind speed over the year (a constant),
- v is the instantaneous wind speed (varies with time), and
- f is the Rayleigh distribution (here it is called the velocity distribution).

MEANING: The integral

$$\int_a^b f(v)\, dv\,, \tag{4}$$

gives the fractional part of the year (between 0 and 1) during which the wind speed ranges between $v = a$ and $v = b$.

We consider a wind farm near my wife's hometown near Sidney, NE where the average annual wind speed is $V = 7.9$ m/s.

Set Digits to 20 at the beginning of your Maple session. ALL results should include proper units!

1. Enter the velocity distribution f as a Maple **function** of v, **not** as an expression. Plot $f(v)$ on the interval $0 \le v \le 30$. Recall that plots should be large enough to show pertinent features but not dominate the page. Then use Maple's `int` command to determine
   (a) how much of the year experiences wind speed ranges from 5 to 12 m/s; Express the result as days/yr, and
   (b) how much of the year experiences wind speeds of 10 m/s or less; Express the result as days/yr.

2. The wind power is therefore

$$P \;=\; 0.6125\, A \int_0^{30} v^3 f(v)\, dv\,. \tag{5}$$

   (a) Use your Simpson's–1/3 program with $n = 12$ subintervals to approximate power P if the turbine diameter is 90 m.
   (b) Repeat Step 2a using $n = 24$ subintervals. Store this result under a descriptive name.

   Express both results in Gigawatts (GW), and store them under descriptive names.

3. We would *expect* the error ratio $\dfrac{\text{Error}(n=12)}{\text{Error}(n=24)}$ to be about what?
   (State the answer in text mode, and use a comprehensible and complete sentence.)

4. In your code use Richardson extrapolation to obtain an improved estimate of the wind power P from the results in Steps 2a and 2b. Again, express the answer in GW, and store it under a descriptive name.

5. Use Maple's `int` command to evaluate (5) to determine the wind power P. Call this result `EXACT`.

6. (a) Use `EXACT` to determine the *percent error* in both approximations in Steps 2a and 2b. Store these under descriptive names.
   (b) Determine the **actual** error ratio $\dfrac{\text{Error}(n=12)}{\text{Error}(n=24)}$. Explain why we don't get the result expected in Step 3.

7. Wind turbines do not convert all available wind energy into usable energy, of course. Using your Richardson result, determine the **annual energy yield** if the turbine is only 26% efficient. Express the annual energy yield in GW-hrs (Gigawatt-hours), and store it under a descriptive name. (There are 8766 hours in a year.)