Due: Friday, February 18

Write the Maple program for **false position** to approximate a zero of a function $f(x)$. **See my Maple example of the bisection method on the course web site.** Also see the posted pseudo-code for false position.

1. The following quantities should be entered in the code:
   - the function f (as a Maple **function**, *not* as a Maple expression),
   - the endpoints a and b of the starting interval,
   - the stopping tolerance TOL, and
   - the maximum number of iterations allowed (this prevents the program from running indefinitely if the tolerance isn't met).
2. At each iteration, the code **must** use the **printf** command (**see Assignment 7**) to print the following on a single line, **in the order shown**:
   - the iteration number n as an integer,
   - the iterate xm in floating point (decimal) form showing 10 decimal places,
   - $|f(xm)|$ in scientific notation showing 4 decimal places.

Set Digits to 20 at the top of your Maple session.

PART I — Test the Code

1. Enter $f = e^{-x} - \cos x$ as a Maple **function** (not an expression), and plot it on the interval $0 \le x \le 8$.
2. Use your code for false position to approximate the first positive zero of the test function

$$f(x) \ = \ e^{-x} - \cos x\,, \tag{1}$$

   by starting on the interval $[1, 4]$. Stop iterating when

$$|f(x)| \ < \ 10^{-5}$$

   at the current iterate. Compare the results with those posted on the web (they should agree).

PART II — Solving an Application Problem

Many processes undergo predictable periodic changes. For examples: the orbits of planets and comets, the 11 year fluctuation in sunspot activity, and the peculiar emergence of the 13–year [*Magicicada neotredecim*] and 17–year [*Magicicada septendecim*] cicadas†.

The number of Pacific salmon often undergoes predictable fluctuations whereby the adult population at the end of year 1 is P_1, the adult population at the end of year 2 is P_2, the adult population at the end of year 3 is P_3, and then the cycle repeats. Thus, the adult population sometimes undergoes 3 year cycles:

$$P_1 \to P_2 \to P_3 \to P_1 \to P_2 \to P_3 \to \cdots$$

†Benjamin Franklin observed and recorded the emergence of cicadas in Pennsylvania in May 1715 and May 1732. What he observed was eventually called Brood X (ten), or the "Great Eastern Brood." The Swedish naturalist Pehr Kalm observed the emergence in PA and NJ again in May 1749 and published his findings in 1756 but rightly credited the earlier observations to Franklin. As predicted, the most recent emergence of Brood X was right on schedule in 2004. It will emerge again in 2021.

By mathematical modeling (using mathematics to model or describe a phenomenon), we may estimate these cyclical values in population. Specifically, we find that populations P_1, P_2, and P_3 are zeros of the function

$$f \;=\; -x - (x-3)\left[\mathrm{e}^{r(x-2)} + \mathrm{e}^{r\left(x-1+(x-3)\mathrm{e}^{r(x-2)}\right)}\right]. \tag{2}$$

where r is a parameter that contains information about the intrinsic birth rate of the population, the death rate due to predation and illness, and the death rate due to cannibalism. We also know from mathematical modeling that P_1, P_2, and P_3 must satisfy

$$P_1 + P_2 + P_3 \;=\; 6\,. \tag{3}$$

Quantities have been scaled mathematically so that results are well-behaved.
Here $\boldsymbol{r} = \mathbf{3.102\ 4399\ 1563\ 5426\ 6095}$.

Set Digits to 20 at the top of your Maple session.

1. Plot f on the interval $0 \le x \le 3$. You'll see that f has four zeros on that interval. However, we are interested only in a set of three zeros whose sum is 6. Namely, we are not interested in the zero at $x = 2$.

   For physical reasons, the only zeros of interest are the zeros of f that are also critical values of f. That means we have a problem in approximating those three zeros! So instead of approximating the zeros of f, we'll approximate the zeros $f'(x)$. But remember, we want only critical values of f at which **both** $f(x) = 0$ and $f'(x) = 0$.

2. To see the zeros of interest more clearly (give each curve a thickness of 5 or 6),

   (a) plot $f(x)$ on the interval $0.0 \le x \le 1.0$ while restricting the vertical range to interval $[0.0, 0.6]$,

   (b) plot $f'(x)$ on the interval $0.0 \le x \le 1.0$ while restricting the vertical range to interval $[-1.0, 2.0]$,

   (c) plot $f(x)$ on the interval $2.5 \le x \le 3.0$ while restricting the vertical range to interval $[-1.5, 0.0]$, and

   (d) plot $f'(x)$ on the interval $2.5 \le x \le 3.0$ while restricting the vertical range to interval $[-10.0, 40.0]$.

In the following, iterate until $|f'(xm)| < 10^{-5}$. Also, since we are approximating a double root, print both $f(xm)$ and $f'(xm)$ at each iterate in scientific notation to 4 decimal places.

3. Use your code for false position to determine the zero of $f'(x)$ using the starting interval $[0.25, 0.5]$. Label this zero $Z1$.

4. Use your code to determine the zero of $f'(x)$ using the starting interval $[2.6, 2.7]$. Label this zero $Z2$.

5. Use your code to determine the zero of $f'(x)$ using the starting interval $[2.97, 2.99]$. Label this zero $Z3$.

6. Calculate $Z1 + Z2 + Z3$. Do you obtain exactly 6 as predicted by Eqn. (3)? **Clearly explain why not in a complete and coherent sentence.**

REMINDERS:

- Remember to use the asterisk $*$ for **all** multiplications. Maple might display the asterisk as a dot.
- When using the **printf** command, all results should be in column form with **decimal points aligned** within each column. **See Assignment 4.**
- Plots should be reasonably sized so as **NOT** to dominate a page. See "Project Requirements" Item 7. Use the `plot` command's `size` option. **See Assignments** $4, 5, 6$**.**
- Only team members who contribute equally and fairly to the entire project should sign the cover page.