Solving 1st Order IVPs **Dr. Kevin G. TeBeest**

$$y' = f(x,y),$$
$$y(x_0) = y_0.$$

	Simple, Low Order Methods	Error
Explicit Euler:	$y_{n+1} = y_n + hf(x_n, y_n)$	$O(h)$
Implicit Euler:	$y_{n+1} = y_n + hf(x_{n+1}, y_{n+1})$	$O(h)$
Trapezoidal Method:	$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_{n+1}, y_{n+1})\right]$	$O(h^2)$

	Modified Euler Method	Error
a) Predictor:	$y^p_{n+1} = y_n + hf(x_n, y_n)$	
b) Corrector:	$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_{n+1}, y^p_{n+1})\right]$	$O(h^2)$

	Classical Runge–Kutta Method	Error
set k_1 :	$k_1 = h\,f(x_n, y_n)$	
set k_2 :	$k_2 = h\,f(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1)$	
set k_3 :	$k_3 = h\,f(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2)$	
set k_4 :	$k_4 = h\,f(x_{n+1}, y_n + k_3)$	
	$y_{n+1} = y_n + \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$	$O(h^4)$

Notes:

1. In each method, the weights (the coefficients that multiply function f) add to 1.
2. The order of accuracy (the exponent on h) equals the number of f evaluations.

