MATH-305 Proper Integrals: Newton–Cotes Formulas

1. Do Problem 322:35.

   (a) Answer: 1.7684
   (b) Answer: 1.7728
   (c) Answer: 1.7904

   Then apply Richardson extrapolation to obtain improved estimates using:

   - results from (a) and (b). Answer: 1.766 933 333
   - results from (b) and (c). Answer: 1.766 933 333
   - results from (a) and (c). Answer: 1.766 933 333

   Why are we getting the same results from Richardson extrapolation in this example?

   For comparison, the exact integral is 1.766 973 094.

2. Repeat problem (1) above using Simpson's–1/3 rule.

   (a) Answer: 1.766 933 333
   (b) Answer: 1.766 933 333
   (c) Answer: 1.767 200 000

   Notice that Simpson's–1/3 rule gave the same answer as we found when using the Trapezoidal rule with Richardson extrapolation. Read Problem 323:46.

3. Function f is given in tabular form as

x	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5
f	2.340000	2.53177	2.73854	2.92973	3.08105	3.17717	3.21337	3.19532

   Use Simpson's–1/3 rule on interval [3.0,5.0] and Simpson's–3/8 rule on interval [5.0,6.5]. Add the results to approximate $\displaystyle\int_{3.0}^{6.5} f(x)\,dx$.

   Answer: 10.228 853

   The exact integral is 10.228 602 672.

4. Repeat Problem 3 above using Simpson's–3/8 rule on interval [3.0,4.5] and Simpson's–1/3 rule on interval [4.5,6.5]. Add the results to approximate $\displaystyle\int_{3.0}^{6.5} f(x)\,dx$.

   Answer: 10.228 802

   The exact integral is 10.228 602 672.

5. What are the sources of error in Problems (3) and (4)?

6.

x	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
f	0.938	1.123	1.345	1.567	1.987	2.212	2.876	2.100	1.532	0.876

   Integrate the above data on the whole interval $[0, 0.9]$ using

   (a) Simpson's–3/8 rule with $h = 0.3$ (ans: 1.7035875)
   (b) Simpson's–3/8 rule with $h = 0.1$ (ans: 1.5598875)
   (c) Richardson extrapolation (ans: 1.5580913)