Ex. Use spherical coords to find

$$\iiint_E e^{(x^2+y^2+z^2)^{3/2}}\,dV$$

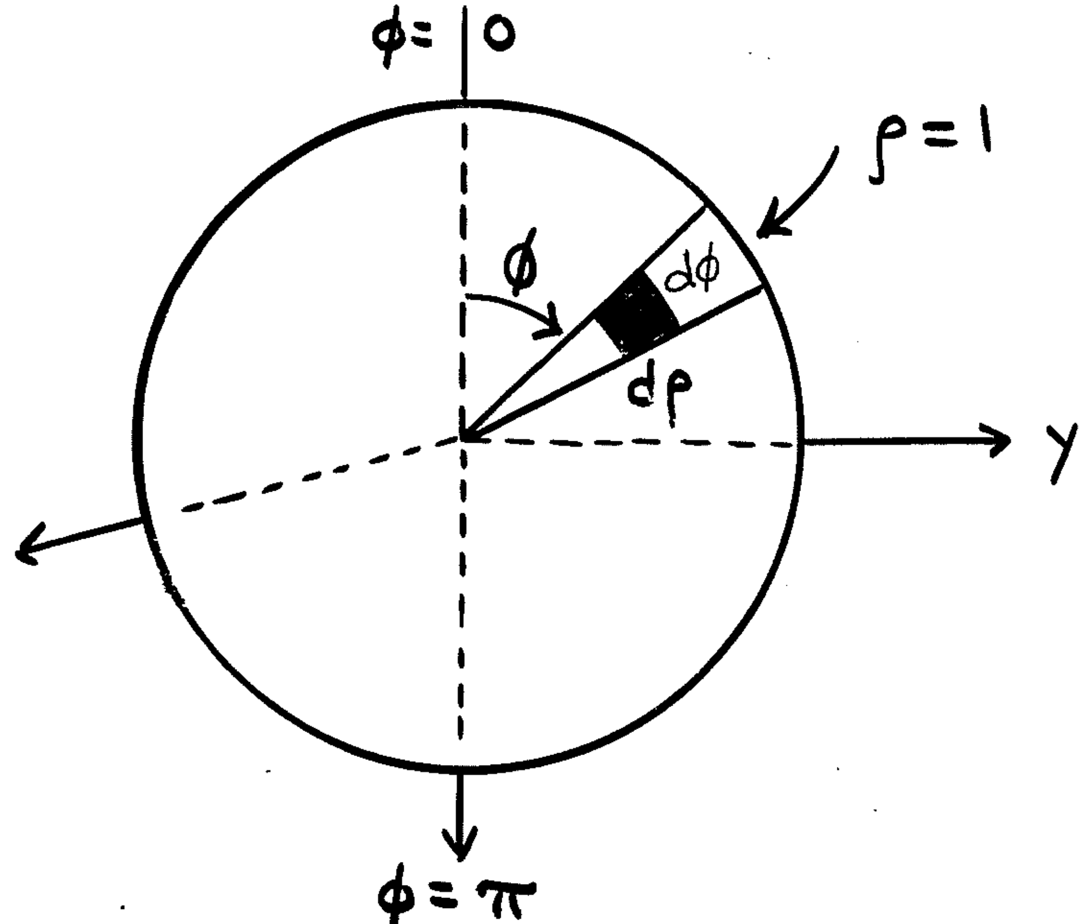
where E is the ball $x^2+y^2+z^2=1$.

In spherical coords,

$$x^2+y^2+z^2 = \rho^2$$

$$dV = \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$$

so E is the ball $\rho = 1$.



Outer: $d\phi$

LL: $\phi = 0$ UL: $\phi = \pi$

View from above:

Middle: $d\theta$

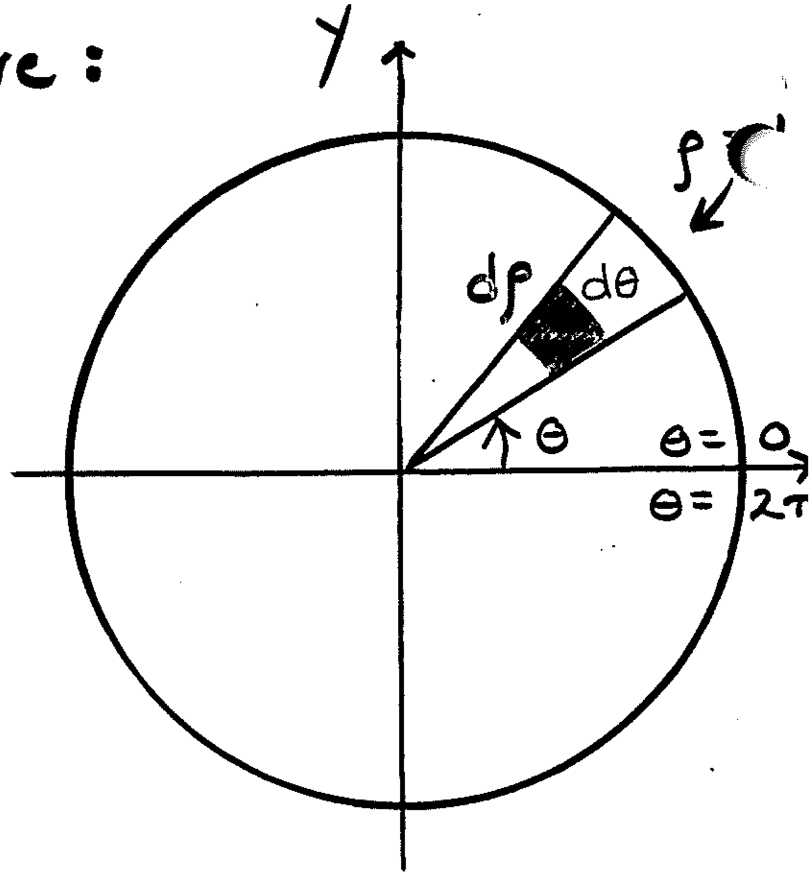
LL: $\theta = 0$

UL: $\theta = 2\pi$

Inner: $d\rho$

LL: $\rho = 0$

UL: $\rho = 1$



Integrand:

$$e^{(x^2+y^2+z^2)^{3/2}} = e^{(\rho^2)^{3/2}} = e^{\rho^3}$$

So

$$\iiint_E e^{(x^2+y^2+z^2)^{3/2}} \, dV$$

$$= \iiint_E e^{\rho^3} \left(\rho^2 \sin\phi \, d\rho \, d\theta \, d\phi \right)$$

$$= \int_0^{\pi} \int_0^{2\pi} \int_0^{1} e^{\rho^3} \cdot \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$$

$$= \int_0^{\pi} \int_0^{2\pi} \sin\phi \int_0^1 \rho^2 e^{\rho^3} \, d\rho \, d\theta \, d\phi$$

$$u = \rho^3 \qquad \frac{du}{d\rho} = 3\rho^2$$

$$d\rho = \frac{du}{3\rho^2}$$

$$\text{LL: } \rho = 0 \Rightarrow u = 0$$

$$\text{UL: } \rho = 1 \Rightarrow u = 1$$

$$= \int_0^{\pi} \int_0^{2\pi} \sin\phi \int_{u=0}^1 \rho^2 e^u \left(\frac{du}{3\rho^2}\right) d\theta \, d\phi$$

$$= \frac{1}{3} \int_0^{\pi} \int_0^{2\pi} \sin\phi \int_{u=0}^1 e^u \, du \, d\theta \, d\phi$$

$$= \frac{1}{3} \int_0^{\pi} \int_0^{2\pi} \sin\phi \cdot e^u \Big|_{u=0}^1 \, d\theta \, d\phi$$

$$= \frac{1}{3} \int_0^{\pi} \int_0^{2\pi} \sin\phi \, (e - e^0) \, d\theta \, d\phi$$

$$= \frac{1}{3}(e - 1) \int_0^{\pi} \sin\phi \int_0^{2\pi} d\theta \, d\phi$$

$$= \frac{1}{3}(e-1) \int_0^{\pi} \sin\phi \cdot \theta \Big|_{\theta=0}^{2\pi} \, d\phi$$

$$= \frac{1}{3}(e-1)\int_0^{\pi} \sin\phi\,(2\pi-0)\,d\phi$$

$$= \frac{1}{3}(e-1)\cdot 2\pi\int_0^{\pi} \sin\phi\,d\phi$$

$$= \frac{2}{3}\pi(e-1)\cdot(-\cos\phi)\Big|_{\phi=0}^{\pi}$$

$$= -\frac{2}{3}\pi(e-1)\cdot(\cos\pi-\cos 0)$$

$$= -\frac{2}{3}\pi(e-1)\cdot\underbrace{(-1-1)}_{-2}$$

$$= +\frac{4}{3}\pi(e-1)$$