DUE: at the beginning of class on Monday, June 13

DO THIS USING MAPLE.

Here we consider fluid flow in the "fully developed region" in a circular pipe of radius a. In the fully developed region, the entrance effects have diminished and the velocity no longer depends on distance from the entrance. Because of viscosity (which produces drag), the fluid near the pipe's center moves faster than that near the pipe wall. Thus, the fluid velocity, called the *velocity profile*, depends only on distance r from the pipe center.

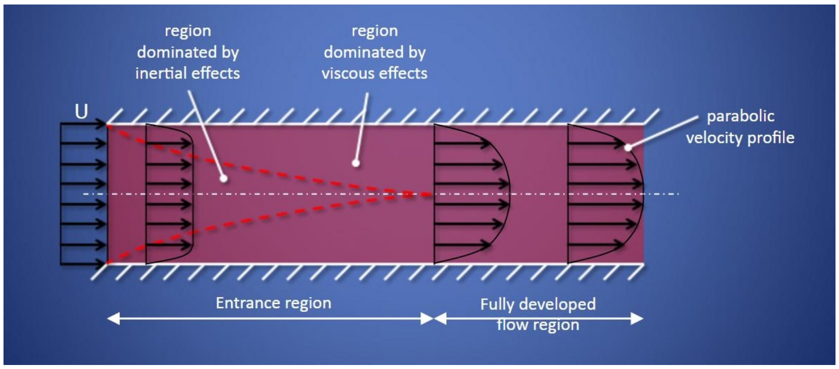

Figure 1: Developing Flow in a Pipe

By solving the Navier–Stokes equations of fluid mechanics, we determine that the velocity profile in the fully developed region is given by

$$V(r) \;=\; \frac{a^2 \Delta P}{4\mu L}\left(1 - \frac{r^2}{a^2}\right), \tag{1}$$

where r is the distance from the pipe center ($0 \le r \le a$), L is the pipe length, ΔP is the pressure change along the pipe length, and μ is the fluid's dynamic viscosity.

1. Show that the fluid velocity at the pipe's wall is 0. This is called the *no-slip boundary condition* (due to drag at the wall).
2. The flow velocity is greatest at the pipe's center. Determine the maximum velocity; name it V_{max}.
3. Now rewrite the formula for $V(r)$ in terms of V_{max}.
4. From fluid mechanics, the volume flow rate Q (volume per unit time) through a circular pipe of radius a is given by

$$Q \;=\; \iint_D V(r)\, dA\,, \tag{2}$$

   where D is a circular pipe cross section of radius a. Use **Maple** to evaluate this integral to determine the volume flow rate Q through the pipe. **Use polar coordinates**.
5. The fluid's average velocity through cross section D is simply the volume flow rate divided by the cross sectional area of D. Determine the fluid's average velocity.

STAPLE ALL PAGES IN THE PROPER ORDER.