Recall that we may approximate a function of two variables, $z = f(x, y)$ using a tangent plane approximation. Namely, we may approximate $f(x, y)$ at points (x, y) that are near a point (x_0, y_0) using

$$f(x,y) \;\approx\; f(x_0,y_0) \;+\; f_x(x_0,y_0)\,\Delta x \;+\; f_y(x_0,y_0)\,\Delta y\,, \tag{1}$$

where $\Delta x = x - x_0$ and $\Delta y = y - y_0$.

This is especially important when we need to approximate a function that is given by tabulated data. In other words, when we do not have a formula for the function.

EXAMPLE: Suppose we have the *apparent temperature* I (in °F) given in tabulated form as a function of the *actual temperature* T (in °F) and the *relative humidity* H.

		% Humidity, H				
		60	65	**70**	75	80
	92	105	108	112	115	119
	94	111	114	**118**	122	127
T	**96**	116	**121**	**125**	**130**	135
	98	123	127	**133**	138	144
	100	129	135	141	147	154

For example, when the actual temperature is 96°F and the humidity is 70%, then the apparent temperature is

$$I(96, 70) \;=\; 125°\text{F}\,.$$

Use linear approximation (tangent plane approximation) to approximate the apparent temperature I when the actual temperature is 97°F and the humidity is 72%.

From Eq. (1), we may approximate this by

$$I(97,72) \;\approx\; I(96,70) \;+\; I_T(96,70)\,\Delta T \;+\; I_H(96,70)\,\Delta H\,. \tag{2}$$

Here,

$$\begin{aligned} \Delta T &= 97 - 96 = +1\,, \\ \Delta H &= 72 - 70 = +2\,. \end{aligned}$$

We also need to approximate the partial derivatives $I_T(96, 70)$ and $I_H(96, 70)$.

(*next page*. . .)

To approximate the partial derivatives $I_T(96,70)$ and $I_H(96,70)$.

		H		
		65	**70**	75
	94	114	**118**	122
T	**96**	**121**	125	**130**
	98	127	**133**	138

1. $I_T(96,70) = \dfrac{\partial I}{\partial T}(96,70)$:

   This is the partial derivative of I *wrt* T, so T varies and H acts as a constant. So we use only numbers from the column $H = 70$. We'll approximate this derivative by calculating the slope between points $(T_1, I_1) = (94, 118)$ and $(T_2, I_2) = (98, 133)$:

$$I_T(96,70) \;=\; \frac{\partial I}{\partial T}(96,70) \;\approx\; \frac{I_2 - I_1}{T_2 - T_1} \;=\; \frac{133-118}{98-94} \;=\; +3.75\,.$$

2. $I_H(96,70) = \dfrac{\partial I}{\partial H}(96,70)$:

   This is the partial derivative of I *wrt* H, so H varies and T acts as a constant. So we use only numbers from the column $T = 96$. We'll approximate this derivative by calculating the slope between points $(H_1, I_1) = (65, 121)$ and $(H_2, I_2) = (75, 130)$:

$$I_H(96,70) \;=\; \frac{\partial I}{\partial H}(96,70) \;\approx\; \frac{I_2 - I_1}{H_2 - H_1} \;=\; \frac{130-121}{75-65} \;=\; +0.9\,.$$

So by Eq. (2),

$$\begin{aligned} I(97,72) \;&\approx\; I(96,70) \;+\; I_T(96,70)\,\Delta T \;+\; I_H(96,70)\,\Delta H \\ &=\; 125 \;+\; (3.75)(1) \;+\; (0.9)(2) \\ &=\; 130.55°\mathrm{F}\,. \end{aligned}$$

CONCLUSION: When the actual temperature is $97°\mathrm{F}$ and the humidity is 72%, the apparent temperature is approximately $130.55°$.

Note 1: Make sure the answer is reasonable. Based on the data in the table, the answer should be between the extreme values $125°\mathrm{F}$ and $133°\mathrm{F}$. Consequently, our result of $130.55°\mathrm{F}$ is reasonable.

Note 2: This process really uses the tangent plane to function $I(T,H)$ at the point $(96,70)$ to approximate $I(97,72)$.