

Ex. Does $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{x^2+y^4}$ exist?

a) try Path C_1 : x-axis:

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{x^2+y^4}$$

$$\stackrel{y=0}{=} \lim_{x \rightarrow 0} \frac{x \cdot 0}{x^2+0}$$

$$= \lim_{x \rightarrow 0} \frac{0}{x^2}$$

$$= \lim_{x \rightarrow 0} 0 = 0$$

So the limit as $(x,y) \rightarrow (0,0)$ along the x-axis is 0.

b) try Path C_2 : the parabola $x = y^2$.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{x^2+y^4}$$

$$\stackrel{x=y^2}{=} \lim_{y \rightarrow 0} \frac{(y^2) \cdot y^2}{(y^2)^2 + y^4}$$

$$= \lim_{y \rightarrow 0} \frac{y^4}{y^4 + y^4}$$

$$= \lim_{y \rightarrow 0} \frac{y^4}{2y^4}$$

$$= \lim_{y \rightarrow 0} \frac{1}{2} = \frac{1}{2} \neq 0$$

$$\text{So } \lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{x^2+y^4}$$

does NOT exist.