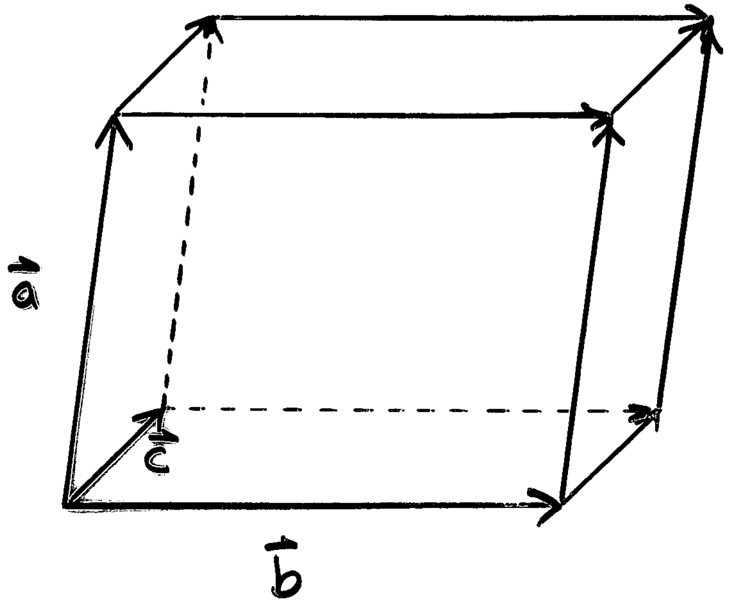
7. The volume of the parallelopiped determined by vectors $\vec{a}$, $\vec{b}$, + $\vec{c}$ is given by the magnitude of the triple scalar product:

$$\text{volume} = |\vec{a} \cdot (\vec{b} \times \vec{c})|$$



Note:

If the triple scalar product of vectors $\vec{a}$, $\vec{b}$, + $\vec{c}$ is 0, then the parallelopiped has 0 volume, so vectors $\vec{a}$, $\vec{b}$, + $\vec{c}$ are coplanar.

Ex. $\vec{a} = \langle 2, 3, -1 \rangle$, $\vec{b} = \langle 5, -2, 9 \rangle$
$\vec{c} = \langle -1, 8, -11 \rangle$

Then

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} 2 & 3 & -1 \\ 5 & -2 & 9 \\ -1 & 8 & -11 \end{vmatrix}$$

$$= 2 \begin{vmatrix} -2 & 9 \\ 8 & -11 \end{vmatrix} - 3 \begin{vmatrix} 5 & 9 \\ -1 & -11 \end{vmatrix} + -1 \begin{vmatrix} 5 & -2 \\ -1 & 8 \end{vmatrix}$$

$$= 2(-50) - 3(-46) - 1(38)$$

$$= 0$$ so $\vec{a}$, $\vec{b}$, + $\vec{c}$ are coplanar