Standard Basis Vectors in Rectangular Coordinates $\mathbf{R}^3$:

1. Def. of $\mathbf{i}$. $\mathbf{i}$ is the unit vector that points in the direction of $+x$: so $\mathbf{i} = \langle 1, 0, 0\rangle$.
2. Def. of $\mathbf{j}$. $\mathbf{j}$ is the unit vector that points in the direction of $+y$: so $\mathbf{j} = \langle 0, 1, 0\rangle$.
3. Def. of $\mathbf{k}$. $\mathbf{k}$ is the unit vector that points in the direction of $+z$: so $\mathbf{k} = \langle 0, 0, 1\rangle$.

Consequently, a vector $\mathbf{a} \in \mathbf{R}^3$ may be written in rect. coords. as $\mathbf{a} = \langle a_1, a_2, a_3\rangle = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$.

Dot Product: The dot product (**inner product**, **scalar product**) of vectors $\mathbf{a}$ and $\mathbf{b}$ is defined as

$$\mathbf{a}\cdot\mathbf{b} \;\equiv\; a_1b_1 + a_2b_2 + a_3b_3\,. \tag{1}$$

Notice that

$$a_1 \;=\; \mathbf{a}\cdot\mathbf{i}\,, \qquad a_2 \;=\; \mathbf{a}\cdot\mathbf{j}\,, \qquad a_3 \;=\; \mathbf{a}\cdot\mathbf{k}\,.$$

Alternatively,

$$\mathbf{a}\cdot\mathbf{b} \;\equiv\; |\mathbf{a}|\,|\mathbf{b}|\cos\theta\,, \tag{2}$$

where θ, $0 \le \theta \le \pi$, is the smallest angle between $\mathbf{a}$ and $\mathbf{b}$. We may use Eq. (2) to determine the smallest angle θ between nonzero vectors $\mathbf{a}$ and $\mathbf{b}$.

Note: The angle θ between nonzero vectors $\mathbf{a}$ and $\mathbf{b}$ is

1. 90° ($\pi/2$ rad) if $\mathbf{a}\cdot\mathbf{b} = 0$ (so $\mathbf{a}$ and $\mathbf{b}$ are **perpendicular (orthogonal)**),
2. acute ($0 \le \theta < \pi/2$) if $\mathbf{a}\cdot\mathbf{b} > 0$,
3. obtuse ($\pi/2 < \theta \le \pi$) if $\mathbf{a}\cdot\mathbf{b} < 0$.

Magnitude ("Length") of a Vector: The magnitude ("length") of vector $\mathbf{a}$ is defined as

$$|\mathbf{a}| \;\equiv\; \sqrt{\mathbf{a}\cdot\mathbf{a}} \;=\; \sqrt{a_1^2 + a_2^2 + a_3^2}\,. \tag{3}$$

Projection: The **vector projection** of $\mathbf{b}$ onto $\mathbf{a}$ is the *vector* given by

$$\text{proj}_{\mathbf{a}}\mathbf{b} \;\equiv\; \frac{\mathbf{b}\cdot\mathbf{a}}{\mathbf{a}\cdot\mathbf{a}}\,\mathbf{a} \;=\; \frac{\mathbf{b}\cdot\mathbf{a}}{|\mathbf{a}|^2}\,\mathbf{a} \;=\; \left(\frac{\mathbf{b}\cdot\mathbf{a}}{|\mathbf{a}|}\right)\frac{\mathbf{a}}{|\mathbf{a}|}\,. \tag{4}$$

Notes:

1. $\text{proj}_{\mathbf{a}}\mathbf{b}$ is the vector part of $\mathbf{b}$ that is *parallel* to $\mathbf{a}$.
2. The vector $\text{perp}_{\mathbf{a}}\mathbf{b} \;\equiv\; \mathbf{b} - \text{proj}_{\mathbf{a}}\mathbf{b}$ is the vector part of $\mathbf{b}$ that is *perpendicular* to $\mathbf{a}$, because
$$\mathbf{b} \;=\; \text{proj}_{\mathbf{a}}\mathbf{b} \;+\; \text{perp}_{\mathbf{a}}\mathbf{b}.$$
3. The component of $\mathbf{b}$ that is parallel to $\mathbf{a}$ (called the **scalar projection** of $\mathbf{b}$ onto $\mathbf{a}$) is the *scalar* $\text{comp}_{\mathbf{a}}\mathbf{b} \;=\; \dfrac{\mathbf{b}\cdot\mathbf{a}}{|\mathbf{a}|}$.
4. Note: $\text{proj}_{\mathbf{a}}\mathbf{b} \;\ne\; \text{proj}_{\mathbf{b}}\mathbf{a}$.

Other Properties: If $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are vectors and c is a scalar, then

1. $\mathbf{u}\cdot\mathbf{v} = \mathbf{v}\cdot\mathbf{u}\,.$
2. $\mathbf{u}\cdot\mathbf{0} = 0\,.$
3. $\mathbf{u}\cdot\mathbf{u} = 0$ if and only if $\mathbf{u} = \mathbf{0}\,$. Otherwise, $\mathbf{u}\cdot\mathbf{u} \;>\; 0\,.$
4. $\mathbf{u}\cdot(\mathbf{v}+\mathbf{w}) \;=\; \mathbf{u}\cdot\mathbf{v} + \mathbf{u}\cdot\mathbf{w}\,.$
5. $c\,(\mathbf{u}\cdot\mathbf{v}) \;=\; (c\,\mathbf{u})\cdot\mathbf{v} \;=\; \mathbf{u}\cdot(c\,\mathbf{v})\,.$