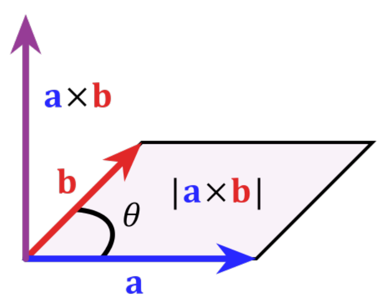
The **cross product** of vectors $\mathbf{a}$ and $\mathbf{b}$ is defined by

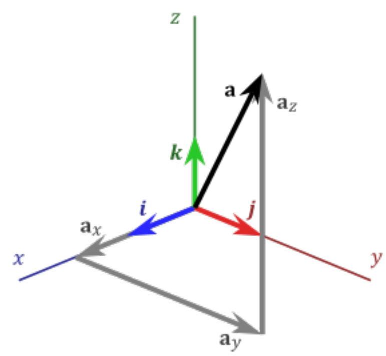
$$\mathbf{a}\times\mathbf{b} \equiv \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = (a_2b_3 - a_3b_2)\mathbf{i} - (a_1b_3 - a_3b_1)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k}\,. \tag{1}$$

The result is a vector $(\mathbf{a}\times\mathbf{b})$ that is perpendicular (orthogonal) to both vectors $\mathbf{a}$ and $\mathbf{b}$. Furthermore,

1. The triplet $\mathbf{a}\to\mathbf{b}\to(\mathbf{a}\times\mathbf{b})$ forms a right-handed system.
2. The area of the parallelogram determined by vectors $\mathbf{a}$ and $\mathbf{b}$ is the magnitude of vector $(\mathbf{a}\times\mathbf{b})$, which may be determined by
$$|\,\mathbf{a}\times\mathbf{b}| = |\mathbf{a}|\,|\mathbf{b}|\sin\theta\,, \tag{2}$$
where θ, $0\le\theta\le\pi$, is the smallest angle between vectors $\mathbf{a}$ and $\mathbf{b}$.

By virtue of Eq. (2), $\mathbf{a}\times\mathbf{b}=\mathbf{0}$ if and only if $\mathbf{a}$ and $\mathbf{b}$ are parallel.





Standard Basis Vectors in $\mathbf{R}^3$: The triplet $\mathbf{i}\to\mathbf{j}\to\mathbf{k}$ forms a right-handed basis for $\mathbf{R}^3$. Furthermore, any cyclical arrangement *also* forms a right-handed basis for $\mathbf{R}^3$, *i.e.*,

$$\mathbf{i}\to\mathbf{j}\to\mathbf{k}\,,\qquad \mathbf{j}\to\mathbf{k}\to\mathbf{i}\,,\qquad \mathbf{k}\to\mathbf{i}\to\mathbf{j}\,.$$

Therefore,

- $\mathbf{i}\times\mathbf{j}=\mathbf{k}$,
- $\mathbf{j}\times\mathbf{k}=\mathbf{i}$, and
- $\mathbf{k}\times\mathbf{i}=\mathbf{j}$.

Noncyclical arrangements do **not** form a right handed basis for $\mathbf{R}^3$, *i.e.*,

$$\mathbf{i}\to\mathbf{k}\to\mathbf{j}\,,\qquad \mathbf{j}\to\mathbf{i}\to\mathbf{k}\,,\qquad \mathbf{k}\to\mathbf{j}\to\mathbf{i}\,.$$

Therefore,

- $\mathbf{j}\times\mathbf{i}=-\mathbf{k}$,
- $\mathbf{k}\times\mathbf{j}=-\mathbf{i}$, and
- $\mathbf{i}\times\mathbf{k}=-\mathbf{j}$.

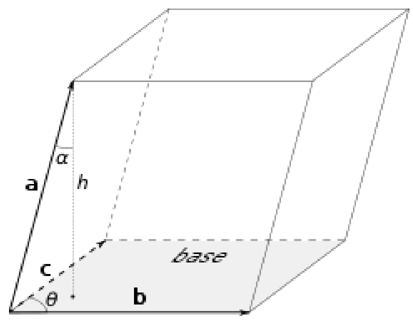
Triple Scalar Product: The **triple scalar product** of vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ is defined by

$$\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) \;\equiv\; \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}. \tag{3}$$

Note that the result is a scalar (not a vector).

Among other things, the magnitude of the triple scalar product gives the volume of the parallelopiped determined by vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$:

$$\text{Volume} \;=\; |\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})|\,. \tag{4}$$



By virtue of Eq. (4), $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = 0$ if and only if vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are co-planar.

Vector Properties: If $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are vectors and k is a scalar, then

1. $\mathbf{a}\times\mathbf{b} \;=\; -\mathbf{b}\times\mathbf{a}$
2. $k\,(\mathbf{a}\times\mathbf{b}) \;=\; (k\,\mathbf{a})\times\mathbf{b} \;=\; \mathbf{a}\times(k\,\mathbf{b})$
3. $\mathbf{a}\times(\mathbf{b}+\mathbf{c}) \;=\; \mathbf{a}\times\mathbf{b}+\mathbf{a}\times\mathbf{c}$
4. $(\mathbf{a}+\mathbf{b})\times\mathbf{c} \;=\; \mathbf{a}\times\mathbf{c}+\mathbf{b}\times\mathbf{c}$
5. $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) \;=\; \mathbf{b}\cdot(\mathbf{c}\times\mathbf{a}) \;=\; \mathbf{c}\cdot(\mathbf{a}\times\mathbf{b})$
6. $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) \;=\; (\mathbf{a}\times\mathbf{b})\cdot\mathbf{c}$
7. $\mathbf{a}\times\mathbf{0} \;=\; \mathbf{0}$
8. $\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) \;=\; (\mathbf{a}\cdot\mathbf{c})\,\mathbf{b}-(\mathbf{a}\cdot\mathbf{b})\,\mathbf{c}$
9. nonzero vectors $\mathbf{a}$ and $\mathbf{b}$ are parallel if and only if $\mathbf{a}\times\mathbf{b} \;=\; \mathbf{0}$