# Chapter 3: Electrical Length

3.1 Given the following equation for the velocity of a signal in a lossy (conductive) medium, determine the equation for the electrical length, *k*, for lossy mediums:

$$v = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]}} = \frac{1}{\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]}}$$

where $\sigma$ is the conductivity, $\mu$ is the permeability, and $\varepsilon$ is the permittivity of the medium. The variable $\beta$ is referred to as the phase constant. For perfect conductors and perfect insulators, what is *k*? Discuss the implications of these results.

3.2S Using a cable catalog where the *L* (inductance) and *C* (capacitance) per unit length and insulation type are provided, determine the effective value of the dielectric constant and compare it to the dielectric constant of the insulation between the conductors for a cable of your choice. (Transmission lines such as coaxial cables where the effective dielectric constant is equal to the insulation dielectric constant should not be selected.) The following definitions may be helpful

$$Z_o = \sqrt{\frac{L}{C}},\quad v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu_{reff}\mu_o\varepsilon_{reff}\varepsilon_o}}$$

where $Z_o$ is the characteristic impedance of and *v* the velocity of propagation in the cable. Compare this effective value to the result obtained by using $\varepsilon_{reff} \approx (\varepsilon_r + 1)/2$. The elements *R* (ohmic resistance) and *G* (dielectric resistance) may be neglected.

3.3S Using a cable catalog where *L* (inductance) and *C* (capacitance) per unit length and insulation type are provided, provide an example where $v \neq 1/\sqrt{LC}$. Explain why this relationship for the velocity is not satisfied.

3.4 Provide a practical example of a conductor (not discussed in this book) that would be considered a radiator. Determine the lowest frequency that this conductor would radiate.

3.5 A three-inch long aluminum wire is attached with glue to an insect and used as part of a locator and tag system. Estimate the lowest frequency that this wire can be used as an antenna.

3.6S By obtaining the dimensions of any real two-conductor cable, determine the highest frequency such that the distance between the conductors is still electrically short.

3.7 An air-core inductor has a length of 1.5 cm and diameter of 0.2 cm. What is the highest frequency that this inductor can still be considered electrically short?

3.8 Why are many antennas used in the VLF band electrically small?

3.9 Why are many antennas used in the UHF band electrically large?

3.10S If the moon is used as a reflector of electrical waves emanating from the earth (to increase the coverage of the signal), what is the smallest frequency that can be transmitted so that the moon is still electrically large? (Ignore the loss and redirection of the signal when passing to the moon.)

3.11 Compare the physical and electrical lengths of a one-quarter wavelength antenna surrounded by free space versus a one-quarter wavelength antenna coated with a very thick good insulator with a dielectric constant of 4. Assume a frequency of operation of 900 MHz in both cases.