Chapter 15: Inductance, Magnetic Coupling, and Transformers

15.1 Determine the total equivalent inductance of the circuit shown in Figure 1.

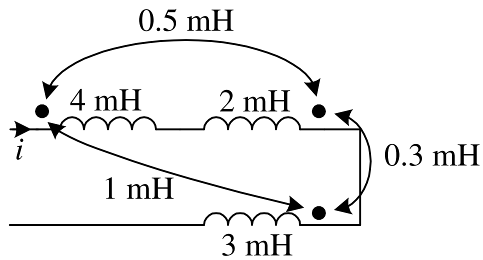

Figure 1

15.2 In reference to the circuit shown in Figure 1, show one possible winding on a toroidal core that would approximately represent this situation. The direction (or orientation) of the windings is important.

15.3 Determine the total equivalent inductance of the circuit shown in Figure 2.

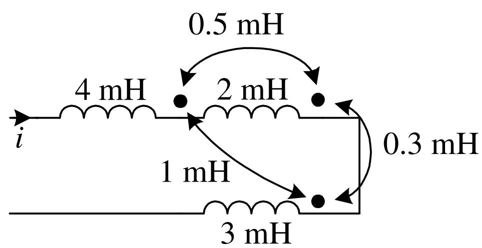

Figure 2

15.4 In reference to the circuit shown in Figure 2, show one possible winding on a toroidal core that would approximately represent this situation. The direction (or orientation) of the windings is important.

15.5 Explain, with an actual winding illustration, how the inductance of a wire-wound resistor can be reduced by using the concept of series-opposing inductors.

15.6C As given in this chapter, the inductance for three, identical long wires (connected in parallel) spaced at the corners of an equilateral triangle with sides of length *d* is

$$L_p = \frac{\mu_o}{2\pi} l_{th} \left\{ \ln\left[\frac{2l_{th}}{\left(r_w d^2\right)^{\frac{1}{3}}} \right] - 0.92 \right\}$$

where l_{th} is the wire length and r_w is the wire radius. Determine the expression for the equivalent inductance of these three wires connected in parallel using the concept of self and mutual partial inductance. Then, numerically compare these two expression using the same parameters and in the same manner as in this multiple conductor grounding strap discussion in this chapter.

15.7C Determine the net impedance of two 20 mil wide copper lands 80 mils apart (center-to-center distance) and four inches in length at f = 100 Hz, 1 kHz, 10 kHz, and 100 kHz. Assume a thickness of 1.38 mils. Determine the width of a single, four-inch long land that results in approximately the same impedance magnitude. Use a numerical package to investigate the variation of the net impedance of the two copper lands with land length and width at these same set of frequencies. Assume the current is uniformly distributed over the traces so that the "thin" inductance expression can be used, as well as the dc resistance expression.

15.8C Two options are available to reduce the overall inductance of a copper land of width w on a PCB. The first option is to increase the width w of the copper land by a factor of two. The second option is to place an additional copper land of width $w/4$ parallel to the copper land of width w. (In this second option the two parallel lands are separated by $3w/4$; thus, the total board usage is identical for both options). Which option is the most effective in reducing the overall inductance of the signal path if w = 20 mil? Explain. Use a numerical package to investigate the variation of the net inductance with land length and width. Ignore the proximity effect.

15.9C Plot the coefficient of coupling versus wire length as in the typical mutual inductance discussion this chapter but use #12, #20, and #28 AWG wire. When is the coefficient of coupling, k, approximately 0.5?

15.10C Numerically explore the effect of two different parallel lead lengths on a capacitor. More specifically, on the same graph plot the net inductance of the two leads when the leads are of the same fixed length, l_{th}, and the net inductance when the length of one lead varies from zero to $2l_{th}$ while the other lead varies from $2l_{th}$ to zero (so that the total lead length remains fixed at $2l_{th}$). Allow l_{th} = 0.25, 0.5, and 1 inch. For which of these lead lengths is the net inductance variation the greatest?

15.11C Repeat Problem 15.10 for two different straight lead lengths on a capacitor.

15.12 In reference to capacitor lead analysis (straight versus spread eagle leads) in this chapter, verify the position of the polarity indicators for both configurations.

15.13C For two #24 AWG wires of length l and m meeting at one point where l = 0.25 inch and m = 0.25, 0.5, 0.75, and 1 inch, plot the partial mutual inductance and total inductance versus the angle between the wires from 0° to 180° in 10° increments. The maximum inductance occurs at what angle? The minimum inductance occurs at what angle?

15.14E Analytically verify that when

$$d \approx \sqrt{2e^{-1} l_{th} r_w} \quad \text{if } l_{th} >> r_w, l_{th} >> d$$

the coefficient of coupling between two straight parallel wires of length l_{th}, separation d, and wire radius r_w is approximately 0.5.

15.15 In reference to "sniffer" discussed in this chapter, if the voltage across the circuit wire is $L\,di/dt$ where L is the self partial inductance of the wire and i is the current through the circuit wire, what is the voltage measured across the open-circuited pickup loop? Assume that M is the mutual partial inductance between the circuit wire and nearest parallel side of the square pickup loop. Assume the same simplifying assumptions as in this chapter discussion. How does the shape of the voltage $L\,di/dt$ across the circuit wire compare to the shape of the open-circuited voltage measured across the pickup loop?

15.16 Provide three examples that clearly show that the magnetic field is not necessarily in the direction of the current producing it. Can the magnetic field and the current producing the field ever be in the same direction?

15.17 Using the flux definition for partial inductance, clearly show L_{p11}, L_{p12}, L_{p21}, and L_{p22} for the two current-carrying wires shown in Figure 3. Show every B_{ij}, dS_i, and I_j.

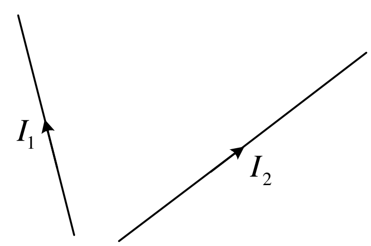

Figure 3

15.18C Determine the transfer function $V_{Ls}(s)/V_{1s}(s)$ for the "sniffer" discussed in this chapter but assume that the length of the circuit wire under test is not equal to width of the "sniffer" probe. Then, plot this transfer function in dB versus frequency if the wire length under test varies from 0.5 to 3 inches. (The mutual inductance will also vary.) Assume that the circuit wire is centered and parallel to the closest side of the loop. In the analytical portion of the analysis, name the self partial inductance of the circuit wire L_c.

15.19 When should the T-equivalent model for the transformer not be used?

15.20EC Determine the transfer function $V_{Ls}(s)/V_{1s}(s)$ for the "sniffer," but assume that a parasitic capacitance exists between the circuit under test and the nearest side of the probe. The T-equivalent model for the transformer may be used.

15.21 In reference to the "sniffer" modeling discussion, if the circuit wire under test is modeled as an inductance of L_1, the sniffer loop is modeled as an inductance of L_2, and the mutual inductance between them is modeled as M, show that well below the cutoff frequency the magnitude of the voltage gain is equal to $M/L_1 = k\sqrt{L_2/L_1}$. By determining the cutoff frequency, show that as L_2 increases, the cutoff frequency decreases.

15.22 An ideal transformer has a turns ratio of 700:40. A 60 Hz, 120 V rms sinusoidal signal source with a source resistance of 10 Ω is connected to the primary high-voltage side while a 100 Ω resistor in series with a 10 mH inductor is connected to the low-voltage side. Determine the current through the 100 Ω resistor (in the time domain), voltage across the inductor (in the frequency domain), and average power absorbed by this resistor and inductor.

15.23 An ideal transformer has a turns ratio of 700:40. A 60 Hz, 120 V rms sinusoidal signal source with a source resistance of 10 Ω is connected to the primary high-voltage side while a 100 Ω resistor in parallel with a 100 μF capacitor is connected to the low-voltage side. By working in the frequency domain, determine the current through the 100 Ω resistor (in the frequency domain), current through the capacitor (in the time domain), and average power absorbed by this load resistor and capacitor.

15.24 For the circuit given in Figure 4, determine the impedance seen by the source, the current I_{lines}, the voltage phasors V_{1s}, V_{2s}, V_{3s},V_{4s}, and the current through the capacitor in the time domain.

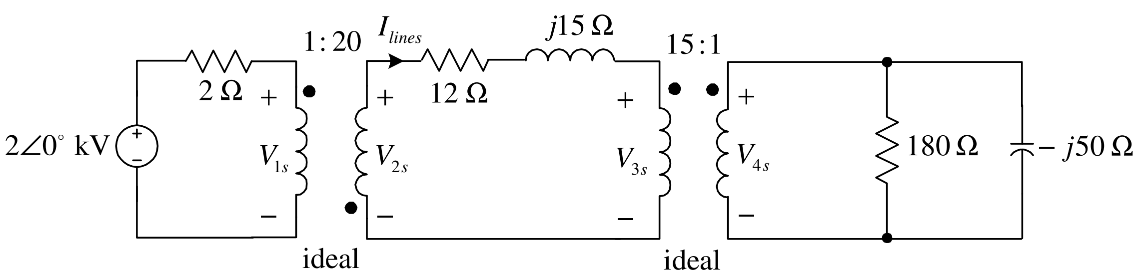

Figure 4

15.25 For the circuit given in Figure 5, determine the impedance seen by the source, the phasor voltage V_{Ls} and its corresponding time-domain voltage $v_L(t)$.

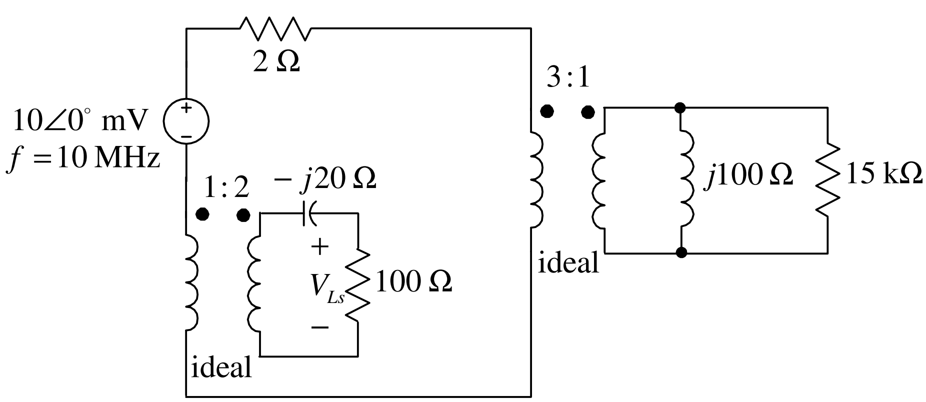

Figure 5

15.26C A speaker is modeled as a 100 pF capacitor that is in parallel with the series combination of a 8 Ω resistor and 10 mH inductor. Determine the minimum integer turns ratio of an ideal transformer to be inserted in parallel with the speaker so that the magnitude of the impedance seen looking into the transformer is less than 1 Ω from 200 Hz to 20 kHz. Plot the magnitude of the speaker's impedance and magnitude of the input impedance of the transformer over this frequency range on the same set of axes.

15.27 For the circuit shown in Figure 6 involving ideal transformers, show that v_3 and v_4 are either the sum or difference of v_1 and v_2. How can this transformer be used for mixing microphone signals? [Tremaine]

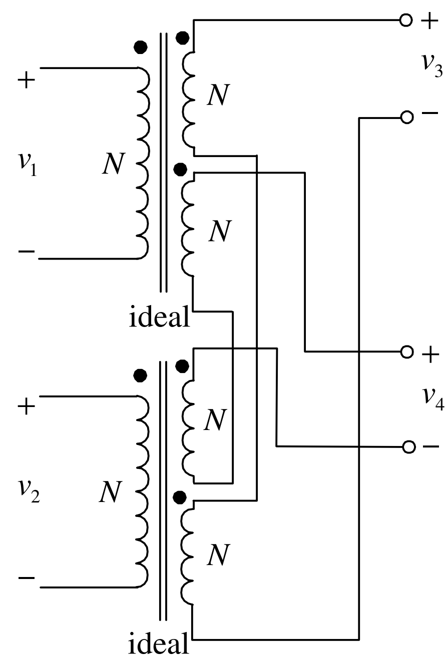

Figure 6

15.28 For the circuit shown in Figure 7 involving one ideal transformer, determine the validity of the following expression for the input impedance:

$$Z_{in} = \frac{N^2 Z_1 + Z_2}{(N-1)^2}$$

Explain in physical terms how this circuit produces infinite input impedance when N =1. What happens if N = 1 and both Z_1 and Z_2 are zero?

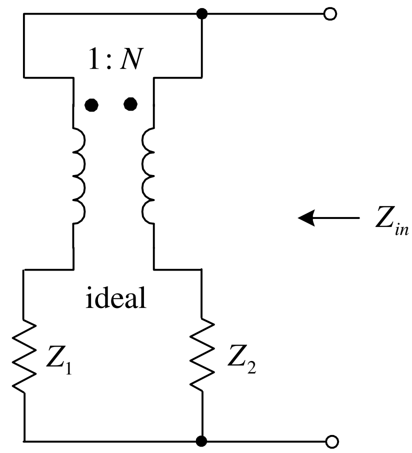

Figure 7

15.29 A lossless linear transformer has a coefficient of coupling of 0.8, primary coil inductance of 100 mH, and secondary coil inductance 700 mH. A 2 mV source voltage at a frequency of 21 MHz with a source resistance of 4 Ω is connected to the low-voltage side of this transformer. A load consisting of a 50 Ω resistor in series with a 10 μH inductor is connected to the high-voltage side of this transformer. Determine the impedance seen by the source and the instantaneous and average power delivered to the load.

15.30C A lossless linear transformer has a coefficient of coupling of k, primary coil inductance of 800 mH, and secondary coil inductance 100 mH. A load consisting of a 50 Ω resistor in series with a 100 μH inductor is connected to the low-voltage side of this transformer. Determine, if it is possible, the value of the coefficient of coupling, k, so that the input impedance from the high-voltage side of the transformer is resonant at 28 MHz.

15.31C For the circuit given in Figure 8, determine the current through the capacitor, $i_C(t)$. Assume the transformer is linear.

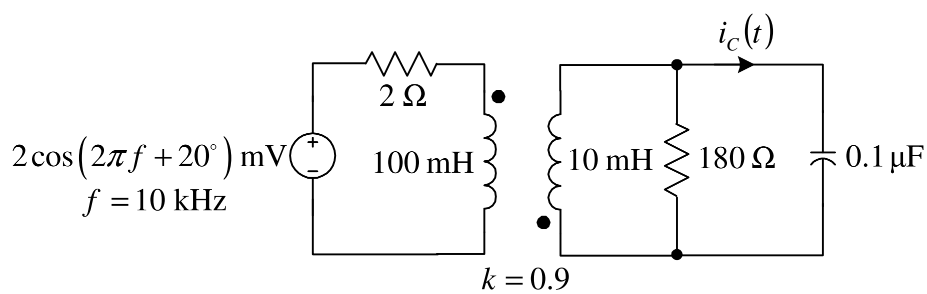

Figure 8

15.32C For the circuit given in Figure 9, determine the current through the inductor, $i_L(t)$. Assume the transformer is linear.

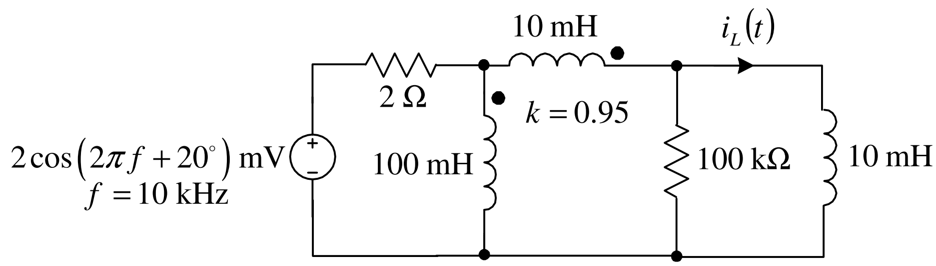

Figure 9

15.33 For the circuit given in Figure 10, write the three mesh-current equations so that the current in any element of the circuit can be determined. However, do not solve these equations. Determine the average power dissipated in R_2 and C in terms of these current variables. Assume linear transformers and sinusoidal steady-state conditions.

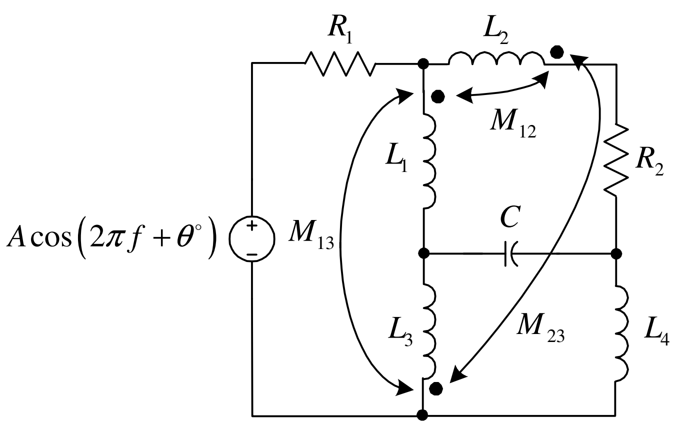

Figure 10

15.34 For the partial circuit shown in Figure 11, determine the time-domain expression for the voltage $v_1(t)$ as a function of the given currents, self inductances, and mutual inductances. The magnetic coupling between the three inductors is linear.

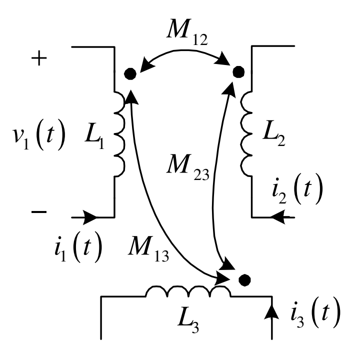

Figure 11

15.35 Referring to Figure 12 and using the expression derived in this chapter for the input impedance looking into the primary side of a linear transformer with perfect coupling and lossless coils, define the reflected impedance as

$$Z_{ref} = \frac{\omega^2 L_1 L_2}{Z_L + j\omega L_2}$$

This is the total impedance reflected into the primary side of the transformer. It does not include the impedance of the primary coil. Starting from this expression for Z_{ref}, verify that the real and imaginary parts of the reflected impedance are equal to, respectively,

$$\operatorname{Re} Z_{ref} = \frac{\omega^4 C_s^2 L_1 L_2 R_L}{\left(\omega^2 L_2 C_s - 1\right)^2 + \omega^2 C_s^2 R_L^2}, \quad \operatorname{Im} Z_{ref} = \frac{-\omega^3 C_s L_1 L_2 \left(\omega^2 L_2 C_s - 1\right)}{\left(\omega^2 L_2 C_s - 1\right)^2 + \omega^2 C_s^2 R_L^2}$$

when the load consists of a resistor R_L in series with a series compensating capacitor C_s. Verify that at the resonant frequency $\omega_o = 1/\sqrt{L_2 C_s}$, where the reflected impedance is entirely real, the real power delivered to the load resistor is

$$P_{avg,R_L} = \frac{\omega_o^2 L_1 L_2}{R_L} I_{1m}^2 = \frac{L_1}{C_s R_L} I_{1m}^2$$

where I_{1m} is the magnitude of the current into the primary coil. Does the average power delivered approach infinity as the load resistance approaches zero? (Be careful!)

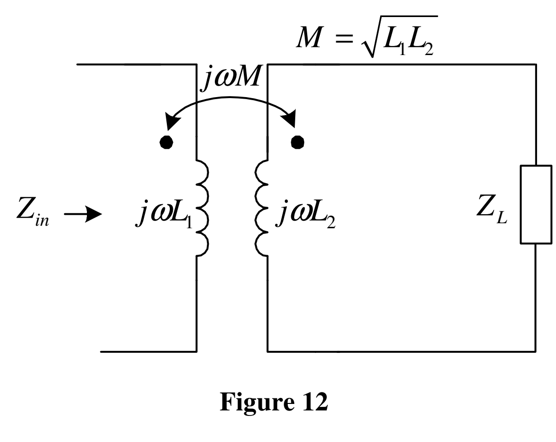

Figure 12

15.36 Referring to Figure 12 and using the expression derived in this chapter for the input impedance looking into the primary side of a linear transformer with perfect coupling and lossless coils, define the reflected impedance as

$$Z_{ref} = \frac{\omega^2 L_1 L_2}{Z_L + j\omega L_2}$$

This is the total impedance reflected into the primary side of the transformer. It does not include the impedance of the primary coil. Starting from this expression for Z_{ref}, verify that the real and imaginary parts of the reflected impedance are equal to, respectively,

$$\operatorname{Re} Z_{ref} = \frac{\omega^2 L_1 L_2 R_L}{R_L^2\left(\omega^2 L_2 C_p - 1\right)^2 + \omega^2 L_2^2}, \quad \operatorname{Im} Z_{ref} = \frac{-\omega^3 L_1 L_2\left[C_p R_L^2\left(\omega^2 L_2 C_p - 1\right) + L_2\right]}{R_L^2\left(\omega^2 L_2 C_p - 1\right)^2 + \omega^2 L_2^2}$$

when the load consists of a resistor R_L in parallel with a parallel compensating capacitor C_p. Verify that at the "secondary resonant" frequency $\omega_o = 1/\sqrt{L_2 C_p}$, the reflected impedance is equal to

$$Z_{ref} = \frac{L_1 R_L}{L_2} - j\frac{L_1}{\sqrt{L_2 C_p}}$$

and the real power delivered to the load resistor is

$$P_{avg,R_L} = \frac{L_1 R_L}{L_2} I_{1m}^2$$

where I_{1m} is the magnitude of the current into the primary coil. Does the average power delivered approach infinity as the load resistance approaches infinity? (Be careful!) At this frequency, how can the reflected reactance be canceled out?

15.37 For a linear transformer with zero-leakage inductance, a primary inductance of L_1, a secondary inductance of L_2, and perfect coupling verify that

$$V_{2s} = V_{1s}\sqrt{\frac{L_2}{L_1}}, \quad I_{1s} = \frac{V_{1s}}{j\omega L_1} - I_{2s}\sqrt{\frac{L_2}{L_1}}$$

Then, shown that these equations essentially describe an ideal transformer with a magnetizing inductance L_1 in parallel with the primary side of the ideal transformer.

15.38 To determine the consequence of using a 60 Hz transformer at other frequencies, first determine the magnitude of the current in the primary and secondary of a linear, lossless 60 Hz transformer (with a primary reactance of 1 Ω, secondary reactance of 20 mΩ, and coefficient of coupling of 0.95) connected to a 1 kΩ load. Then, determine these currents if the frequency is 25 Hz (an old power frequency). Then, determine these currents at 400 Hz (a power frequency used on some aircraft because of the reduced size of the transformers). For which of these two non-60 Hz frequencies is it probably safest to use this 60 Hz transformer?

15.39 Determine the current, voltage, and impedance relationships for a four-winding transformer (three separate windings on the secondary) as was done for the three-winding transformer given in this chapter.

15.40 Determine all possible impedance transformations for the two-tapped transformer given in Figure 13 if R_L = 15 kΩ, N_1 = 100, N_2 = 10, N_3 = 13, and N_4 = 20.

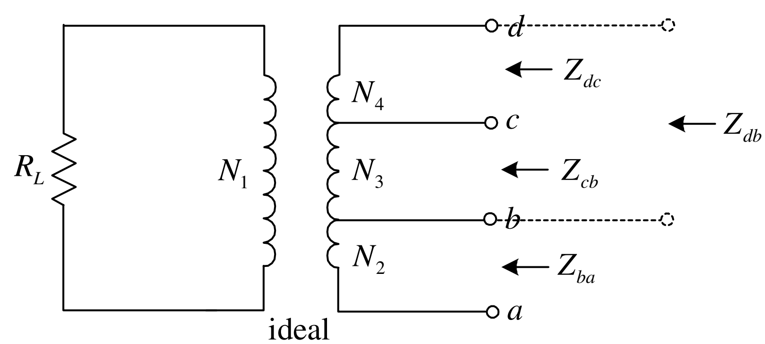

Figure 13

15.41 Determine all possible impedance transformations for a three-tapped transformer analogous to the one given in Figure 13 if R_L = 15 kΩ, N_1 = 100, N_2 = 5, N_3 = 10, N_4 = 25, and N_5 = 20.

15.42 One 120 V to 10 V transformer and one 240 V to 24 V transformer are available. Sketch all possible connections between these two transformers and label all input and output voltages. Clearly show the dot indicators for each connection.

15.43 One 120 V to 15 V transformer is rated at 200 VA and another 120 V to 15 V transformer is rated at 100 VA. Determine the ratings associated with all series and parallel combinations of the primaries and secondaries of these two transformers. As one example, if the primaries are connected in series (adding) and the secondaries in connected in parallel what is the rating of the combination?

15.44 By repeating the analysis given in this chapter for port 1 of the three-port hybrid transformer, show that if v_2 is turned on and v_1 and v_4 are turned off that i_x is zero when

$$R_3 = \frac{R_4}{\left(\frac{N_z}{N_x}\right)^2}$$

and the output voltage is

$$v_z = -v_2 \frac{\frac{R_4}{\left(\frac{N_z}{N_x}\right)^2}}{R_2 + \frac{2R_4}{\left(\frac{N_z}{N_x}\right)^2}} \frac{N_x}{N_z}$$

15.45 Derive the following current expressions provided in this chapter for the three-port hybrid transformer when only v_4 is activated:

$$i_x = -v_4 \frac{\frac{N_z}{N_x}}{2R_4 + \frac{R_1}{\left(\frac{N_x}{N_z}\right)^2}}, \quad i_y = -v_4 \frac{\frac{N_z}{N_x}}{2R_4 + \frac{R_2}{\left(\frac{N_x}{N_z}\right)^2}}$$

15.46 By writing all of the necessary expressions as with the three-port analysis in this chapter, verify that for a four-port hybrid transformer with v_1, v_2, and v_4 turned off that the voltage across resistors R_1 and R_2 (if $R_1 = R_2$) is

$$v_3 \frac{R_1}{R_1 + 2R_3}$$

Do not use symmetry to determine any of the voltages or currents.

15.47C Derive the expressions given in Figure 14 for both load voltages.

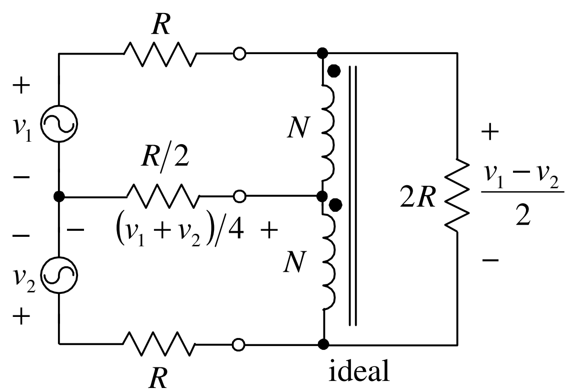

Figure 14

15.48C Derive the expressions given in Figure 15 for both load voltages.

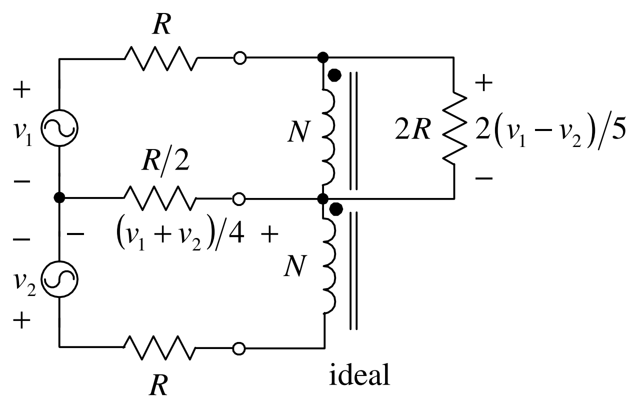

Figure 15

15.49EC Explain how the circuit given in Figure 16 can act as a sound mixer. Specifically, determine the necessary conditions on the various resistors and turns ratios so that when v_1 is turned on and all other voltage sources turned off that the current through R_2 is zero and the currents through R_6 and R_7 are small in comparison to the current through R_1. [Tremaine]

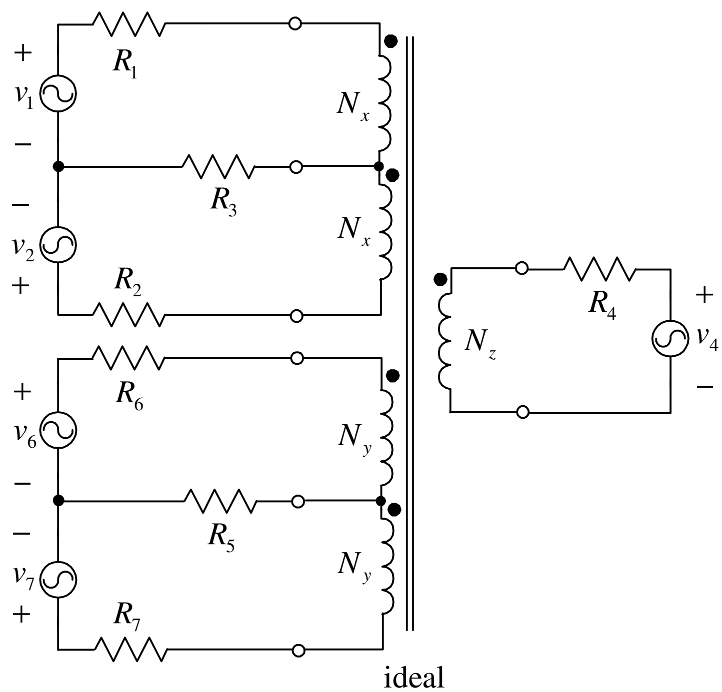

Figure 16

15.50 Repeat the entire autotransformer analysis given in this chapter, including both impedance transformation equations, for the autotransformer shown in Figure 17.

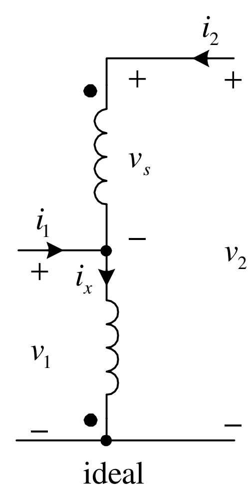

Figure 17

15.51 Repeat the entire autotransformer analysis given in this chapter, including both impedance transformation equations, for the autotransformer shown in Figure 18.

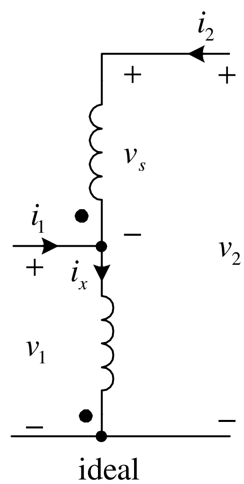

Figure 18

15.52 Determine the ratio of the short-circuit current of an ideal autotransformer to the short-circuit current of an ideal ordinary transformer. Which is larger?

15.53 An autotransformer is used to supply a 1,200 VA load at 1,000 V rms with a 0.95 power factor from a 900 V rms source. Determine the turns ratio of the series to the common winding, the current (magnitude and phase) in each winding, and the voltage (magnitude and phase) across each winding assuming the autotransformer is ideal. Then, determine the voltage ratio and power rating (VA) of this autotransformer if it was connected as an ordinary (nonauto) transformer.

15.54 For the three-winding autotransformer shown in Figure 19, determine the voltage across each load, current (magnitude and phase) through each load, and current (magnitude and phase) through each winding if

$$Z_{L1} = 30 + j5,\ Z_{L2} = 15 - j3$$

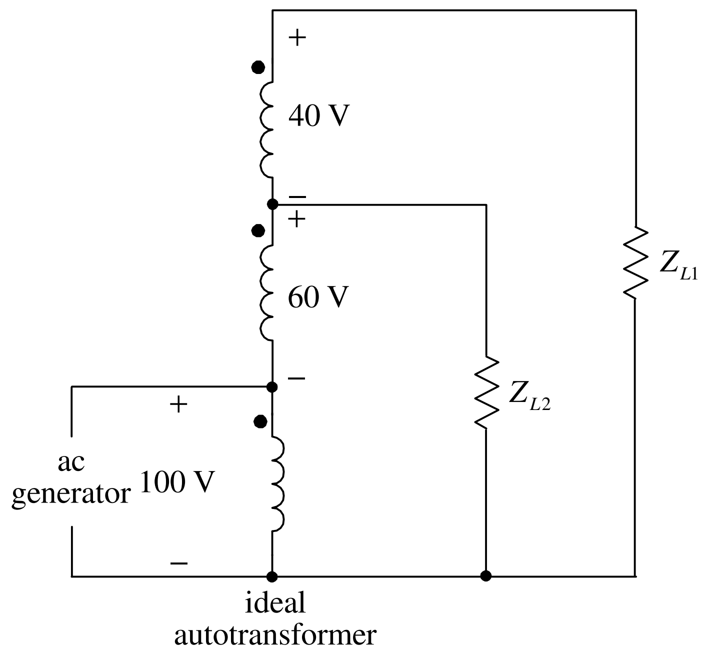

Figure 19

15.55 Determine the expression for the series and common coil current through a step-up autotransformer assuming that the inductance of the common winding is L_1, the inductance of the series winding is L_s, the coefficient of coupling between the coils is k, and the load is purely resistive. Determine the ratio of the current through the series winding to the current in the common winding. Determine this ratio if $L_1 >> L_s$. Determine this ratio if $L_1 << L_s$. Determine this ratio if $L_1 = L_s$.

15.56 Determine the expression for the series and common coil current through a step-down autotransformer assuming that the inductance of the common winding is L_1, the inductance of the series winding is L_s, the coefficient of coupling between the coils is k, and the load is purely resistive. Determine the ratio of the current through the series winding to the current in the common winding. Determine this ratio if $L_1 >> L_s$. Determine this ratio if $L_1 << L_s$. Determine this ratio if $L_1 = L_s$.

15.57 Determine whether the coefficient of coupling for a linear transformer can be determined from the expression

$$k = \sqrt{1 - \frac{C_{oc}}{C_{sc}}}$$

The variable C_{oc} is the series capacitance added across the primary of an open-circuited secondary that will have some convenient resonant frequency, and C_{sc} is the added series capacitance across the primary of short-circuited secondary that will have the same resonant frequency. [Valley]

15.58EC After obtaining the new voltage gain transfer function, repeat the entire numerical analysis given for the double-tuned transformers in this chapter if one capacitor is in parallel with the primary winding while the other capacitor is in series with the secondary winding.

15.59C Derive the expressions for the primary and secondary currents for a perfect linear transformer ($k = 1$) for an impulsive, current input source. Do not use source transformations. Assume that the load is purely resistive and the primary side contains a resistance in parallel with the current source. For the parameters used in the step voltage function version in this chapter, plot the primary and secondary currents. What happens when the load resistance is infinite, corresponding to an unloaded secondary?

15.60C Derive the expressions for the primary and secondary currents for a perfect linear transformer ($k = 1$) for step, current input source. Do not use source transformations. Assume that the load is purely resistive and the primary side contains a resistance in series with the current source. For the parameters used in the step voltage function version in this chapter, plot the primary and secondary currents. What happens when the load resistance is infinite, corresponding to an unloaded secondary?

15.61C Derive the expressions for the primary and secondary currents for a linear transformer ($k \neq 1$) for an impulsive, current input source. Do not use source transformations. Assume that the load is purely resistive and the primary side contains a resistance in series with the current source. For the parameters used in the step voltage function version in this chapter, plot the primary and secondary currents. What happens when the load resistance is infinite, corresponding to an unloaded secondary?

15.62C Derive the expressions for the primary and secondary currents for a linear transformer ($k \neq 1$) for step, current input source. Do not use source transformations. Assume that the load is purely resistive and the primary side contains a resistance in series with the current source. For the parameters used in the step voltage function version in this chapter, plot the primary and secondary currents. What happens to the secondary voltage when the load resistance is infinite, corresponding to an unloaded secondary?