**Problem 1.** Solve the equation $\log_x(x+2) = 2$.

**Problem 2.** Solve the inequality:

$$0.5^{|x|} > 0.5^{x^2}.$$

**Problem 3.** The integers from 1 to 2015 are written on the blackboard. Two randomly chosen numbers are erased and replaced by their differense giving a sequence with one less number. This process is repeated until there is only one number remaining. Is the remaining number even or odd? Justify your answer.

**Problem 4.** Four circles are constructed with the sides of a convex quadrilateral as the diameters. Does there exist a point inside the quadrilateral that is not inside the circles? Justify your answer.

**Problem 5.** Prove that for any finite sequence of digits there exists an integer the square of which begins with that sequence.

**Problem 6.** The distance from the point $P$ to two vertices $A$ and $B$ of an equilateral triangle are $|PA| = 2$ and $|PB| = 3$. Find the greatest possible value of $|PC|$.

**Solutions**

**Problem 1.**

$$x+2=x^2$$
$$x^2-x-2=0$$
$$(x-2)(x+1)=0$$
$$x=2 \text{ or } x=-1$$

$x=-1$ is a false solution.

$$\textbf{Answer}: \quad x=2$$

**Problem 2.**

$$|x|<x^2$$

**The first case:** $x \geq 0$.

$$x<x^2$$
$$x^2-x>0$$
$$x>1$$

**The second case:** $x<0$.

$$-x<x^2$$
$$x^2+x>0$$
$$x<-1$$

Answer: $(-\infty,-1)\cup(1,+\infty)$.

**Problem 3.**

The sequence of integers from 1 to 2015 contains 1007 even numbers and 1008 odd numbers.

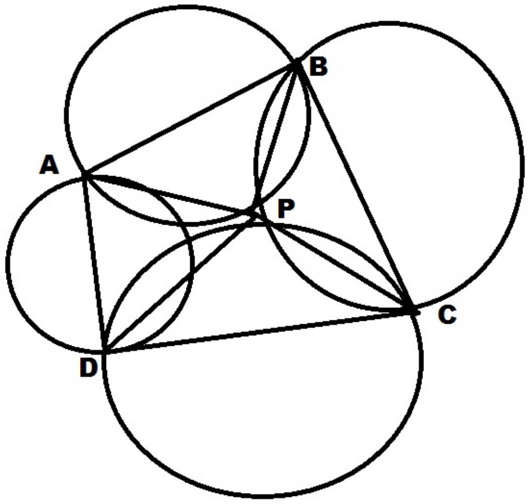
Consider 3 cases:

1) If both erased numbers are even they are replaced by an even number. So the number of odd numbers does not change.

2) If one number is even and another one is odd they are replaced by an odd number. So the number of odd numbers does not change.

3) If both erased numbers are odd they are replaced by an even number. So the number of odd numbers decreases by two.

Thus at every step the number of odd numbers either remains the same or decreases by two. Since originally it was even, it will remain even. Therefore, when there is one number remaining this number is even.




**Problem 4.**

Let us assume that such a point exists. Denote it by $P$. Let $A$, $B$, $C$, and $D$ be verteces of the quadrilateral. Then each one of the angles $APB$, $BPC$, $CPD$, $DPA$ is less than $90^o$ since they are outside of the corresponding circles. So the sum of these angles is less than $360^o$. However, for a convex quadrilateral it is supposed to be $360^o$. This contradiction proves that such point does not exist.

**Problem 5.**

Let the given digits be $a_1a_2\dots a_n$. Let $a$ be the number with thse digits. Consider two numbers:

$$N_1 = a \cdot 10^{3n} = a_1a_2\dots a_n0\dots 0,$$

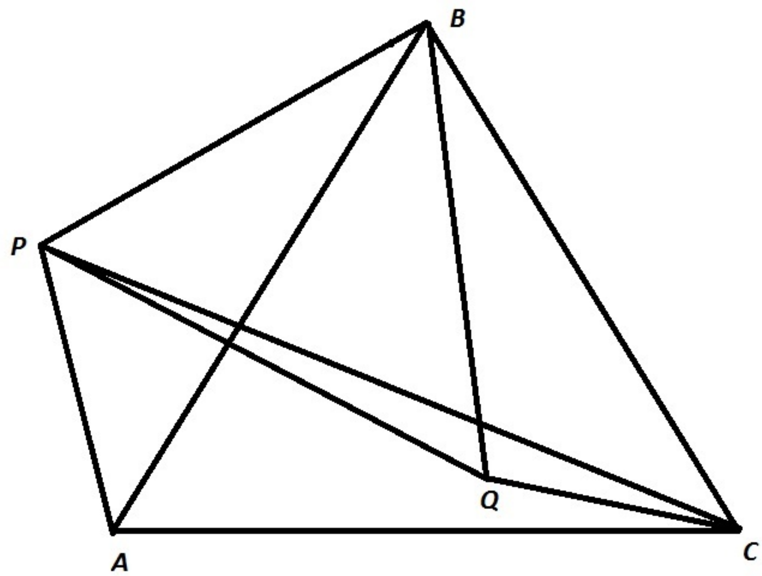
$$N_2 = a \cdot 10^{3n} + (10^{3n+1} - 1) = a_1a_2\dots a_n9\dots 9.$$

Denote by $m$ the greatest integer for which $m^2 \le N_1$. Since $N_1 \le 10^{4n}$, $m \le 10^{2n}$. Therefore

$$N_1 < (m+1)^2 = m^2 + 2m + 1 < N_1 + 2 \cdot 10^{2n} + 1 <$$

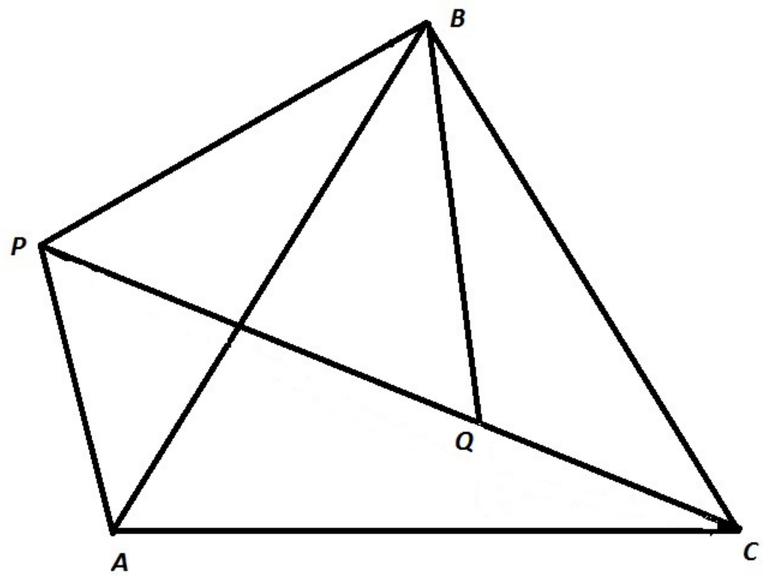
$$< N_1 + 10^{3n} - 1 = N_2.$$

Since $(m+1)^2$ is between $N_1$ and $N_2$ its first digits are $a_1a_2\dots a_n$.




**Problem 6.**

Draw a segment $BQ$ such that $\angle CBQ = \angle PBA$ and $|BQ| = |BP|$ and connect the point $Q$ with the points $P$ and $C$.
Since $\angle PBQ = \angle ABC = 60^o$ and $|BQ| = |BP|$, the triangle $BPQ$ is equilateral. Therefore, $|PQ| = |PB| = 3$.
Since $|BP| = |BQ|$, $|AB| = |BC|$, and $\angle PBA = \angle QBC$, triangles $PBA$ and $QBC$ are equal. Therefore, $|QC| = |PA| = 2$.
In the triangle $PQC$, $|PC| \leq |PQ| + |QC| = |PB| + |PA| = 5$.
$|PC|$ achieves its greatest value 5 when point $Q$ belongs to the segment $PC$.




To show that it is possible choose any point $B$ such that $|PB| = 3$. The construct an equilateral triangle $PBQ$. Then choose point $C$ such that $Q$ belongs to $PC$ and $|QC| = 2$.
Choose point $A$ such that $ABC$ is an equilateral triangle. Since by construction the triangles $PBA$ and $QBC$ are equal $|PA| = 2$.

**Answer:** The greatest possible value of $|PC|$ is 5.