

Kettering University Mathematics Olympiad For High School Students 2012

1. Solve the equation

$$\frac{\sqrt{x^2 - 2x + 1}}{x^2 - 1} + \frac{x^2 - 1}{\sqrt{x^2 - 2x + 1}} = \frac{5}{2}.$$

2. Solve the inequality:

$$\frac{1 - 2\sqrt{1 - x^2}}{x} \leq 1.$$

3. Let $ABCD$ be a convex quadrilateral such that the length of the segment connecting midpoints of the two opposite sides AB and CD equals $\frac{|AD| + |BC|}{2}$. Prove that AD is parallel to BC .

4. Solve the equation:

$$\frac{1}{\cos x} + \frac{1}{\sin x} = 2\sqrt{2}.$$

5. Long, long ago, far, far away there existed the Old Republic Galaxy with a large number of stars. It was known that for any four stars in the galaxy there existed a point in space such that the distance from that point to any of these four stars was less than or equal to R . Master Yoda asked Luke Skywalker the following question: Must there exist a point P in the galaxy such that all stars in the galaxy are within a distance R of the point P ? Give a justified argument that will help Like answer Master Yoda's question.
6. The Old Republic contained an odd number of inhabited planets. Some pairs of planets were connected to each other by space flights of the Trade Federation, and some pairs of planets were not connected. Every inhabited planet had at least one connections to some other inhabited planet. Luke knew that if two planets had a common connection (they are connected to the same planet), then they have a different number of total connections. Master Yoda asked Luke if there must exist a planet that has exactly two connections. Give a justified argument that will help Luke answer Master Yoda's question.