

Kettering University Mathematics Olympiad For High School Students 2010

1. Find the value of the parameter a for which the following system of equations does not have solutions:

$$\begin{cases} ax + 2y = 1 \\ 2x + ay = 1. \end{cases}$$

2. Find all solutions of the equation

$$\cos(2x) - 3\sin(x) + 1 = 0.$$

3. A circle of a radius r is inscribed into a triangle. Tangent lines to this circle parallel to the sides of the triangle cut out three smaller triangles. The radiuses of the circles inscribed in these smaller triangles are equal to 1, 2 and 3. Find r .
4. Does there exist an integer k such that $\log_{10}(1 + 49367 \cdot k)$ is also an integer?
5. A plane is divided by 3015 straight lines such that neither two of them are parallel and neither three of them intersect at one point. Prove that among the pieces of the plane obtained as a result of such division there are at least 2010 triangular pieces.