

Kettering University Mathematics Olympiad For High School Students 2009

1. Prove that if a, b, c, d are real numbers, then

$$\max\{a + c, b + d\} \leq \max\{a, b\} + \max\{c, d\}$$

2. Find the smallest positive integer whose digits are all ones which is divisible by 333333.
3. Find all integer solutions of the equation

$$\sqrt{x} + \sqrt{y} = \sqrt{2560}.$$

4. Find the irrational number:

$$A = \sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2} + \cdots \frac{1}{2}\sqrt{\frac{1}{2}}}}}$$

(n square roots).

5. The Math country has the shape of a regular polygon with N vertexes. N airports are located on the vertexes of that polygon, one airport on each vertex. The Math Airlines company decided to build K additional new airports inside the polygon. However the company has the following policies: (i) it does not allow three airports to lie on a straight line, (ii) any new airport with any two old airports should form an isosceles triangle. How many airports can be added to the original N ?
6. The area of the union of the n circles is greater than $9m^2$ (some circles may have non-empty intersections). Is it possible to choose from these n circles some number of non-intersecting circles with total area greater than $1m^2$?