

**Kettering University Mathematics Olympiad For High School Students 2007**

1. An airplane travels between two cities. The first half of the distance between the cities is traveled at a constant speed of 600 mi/hour, and the second half of the distance is traveled at a constant speed of 900mi/hour. Find the average speed of the plane.
2. The figure below shows two egg cartons, A and B. Carton A has 6 spaces (cell) and has 3 eggs. Carton B has 12 cells and 3 eggs. Two cells from the total of 18 cells are selected at random and the contents of the selected cells are interchanged. (Not that one or both of the selected cells may be empty.)

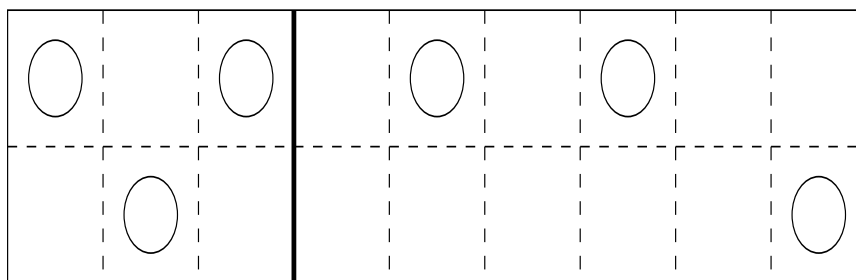


Figure 1: Diagram for Problem 2

- (a) Find the number of selections/interchanges that produce a decrease in the number of eggs in carton A- leaving carton A with 2 eggs.
- (b) Assume that the total number of eggs in cartons A and B is 6. How many eggs must initially be in carton A and in carton B so that the number of selections/interchanges that lead to an increase in the number of eggs in A equals the number of selections/interchanges that lead to an increase in the number of eggs in B.
  - In other words, find the initial distribution of 6 eggs between A and B so that the likelihood of an increase in A equals the likelihood of an increase in B as the result of a selection/interchange. Prove your answer.

3. Divide the following figure into four equal parts (parts should be of the same shape and of the same size, they may be rotated by different angles however they may not be disjoint and reconnected).

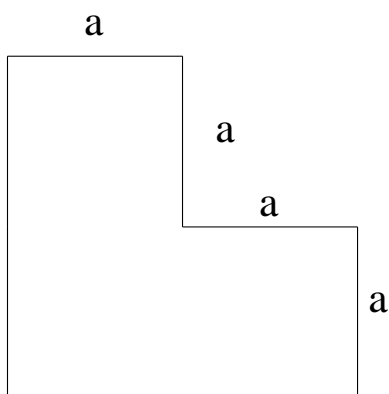


Figure 2: Diagram for Problem 3

4. Find the **exact** numerical value of  $\sqrt[3]{5\sqrt{2} + 7} - \sqrt[3]{5\sqrt{2} - 7}$  (do not use a calculator and do not use approximations).
5. The medians of a triangle have length 9,12 and 15cm respectively. Find the area of the triangle.
6. The numbers  $1, 2, 3, \dots, 82$  are written in an arbitrary order. Prove that it is possible to cross out 72 numbers in such a way the remaining number will be either in increasing order or in decreasing order.