

Kettering University Mathematics Olympiad For High School Students 2004

1. Find **all** real solutions of the system

$$\begin{aligned}x^5 + y^5 &= 1 \\x^6 + y^6 &= 1.\end{aligned}$$

2. The centers of three circles of the radius R are located in the vertexes of equilateral triangle. The length of the sides of the triangle is a and $\frac{a}{2} < R < a$. Find the distances between the intersection points of the circles, which are outside of the triangle.
3. Prove that no positive integer power of 2 ends with four equal digits.
4. A circle is divided in 10 sectors. 90 coins are located in these sectors, 9 coins in each sector. At every move you can move a coin from a sector to one of two neighbor sectors. (Two sectors are called neighbor if they are adjoined along a segment.) Is it possible to move all coins into one sector in exactly 2004 moves?
5. Inside a convex polygon several points are arbitrary chosen. Is it possible to divide the polygon into smaller convex polygons such that every one contains exactly one given point? Justify your answer.
6. A troll tried to spoil a white and red 8x8 chessboard. The area of every square of the chessboard is one square foot. He randomly painted 1.5% of the area of every square with black ink. A grasshopper jumped on the spoiled chessboard. The length of the jump of the grasshopper is exactly one foot and at every jump only one point of the chessboard is touched. Is it possible for the grasshopper to visit every square of the chessboard without touching any black point? Justify your answer.