Kettering University Mathematics Olympiad For High School Students 2001

Problem 1. Find the largest k such that the equation $x^2 - 2x + k = 0$ has at least one real root.

Problem 2. Indiana Jones needs to cross a flimsy rope bridge over a mile long gorge. It is so dark that it is impossible to cross the bridge without a flashlight. Furthermore, the bridge is so weak that it can only support the weight of two people. The party has only one flashlight, which has a weak beam so whenever two people cross, they are constrained to walk together, at the speed of the *slower* person. Indiana Jones can cross the bridge in 5 minutes. His girlfriend can cross in 10 minutes. His father needs 20 minutes, and his father's side kick needs 25 minutes. They need to get everyone across safely in on hour to escape the bay guys. Can they do it?

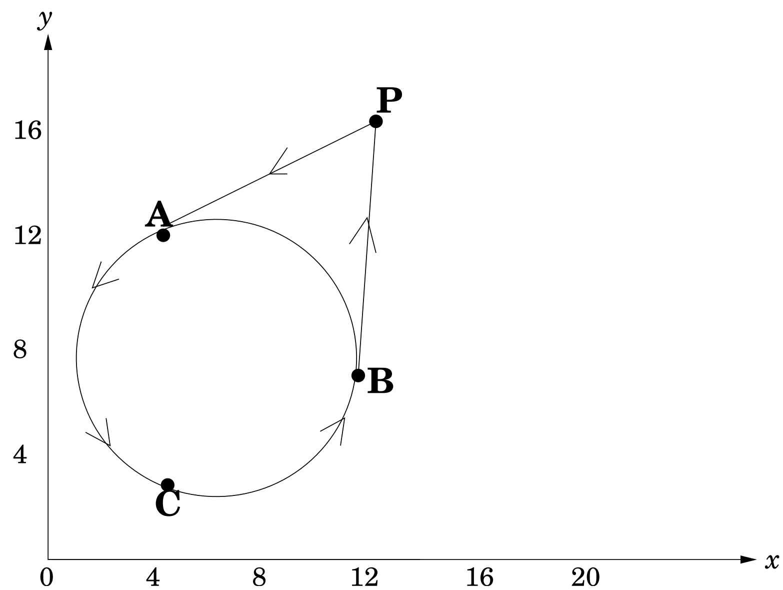
Problem 3. There are ten big bags with coins. Nine of them contain fare coins weighing 10g. each, and one contains counterfeit coins weighing 9g. each. By one weighing on a digital scale find the bag with counterfeit coins.

Problem 4. Solve the equation:

$$\sqrt{x^2 + 4x + 4} = x^2 + 5x + 5.$$

Problem 5. (a) In the $x - y$ plane, analytically determine the length of the path $P \longrightarrow A \longrightarrow C \longrightarrow B \longrightarrow P$ around the circle $(x-6)^2 + (y-8)^2 = 25$ from the point $P(12, 16)$ to itself.

(b) Determine coordinates of the points A and B.



Problem 6. (a) Let $ABCD$ be a convex quadrilateral (it means that diagonals are inside the quadrilateral). Prove that

$$\text{Area}(ABCD) \le \frac{|AB| \cdot |AD| + |BC| \cdot |CD|}{2}.$$

(b) Let $ABCD$ be an arbitrary quadrilateral (not necessary convex). Prove the same inequality as in part (a).

(c) For an arbitrary quadrilateral $ABCD$ prove that

$$\text{Area}(ABCD) \le \frac{|AB| \cdot |CD| + |BC| \cdot |AD|}{2}.$$