Module 3. Gas Turbine Power Cycles – Brayton Cycle

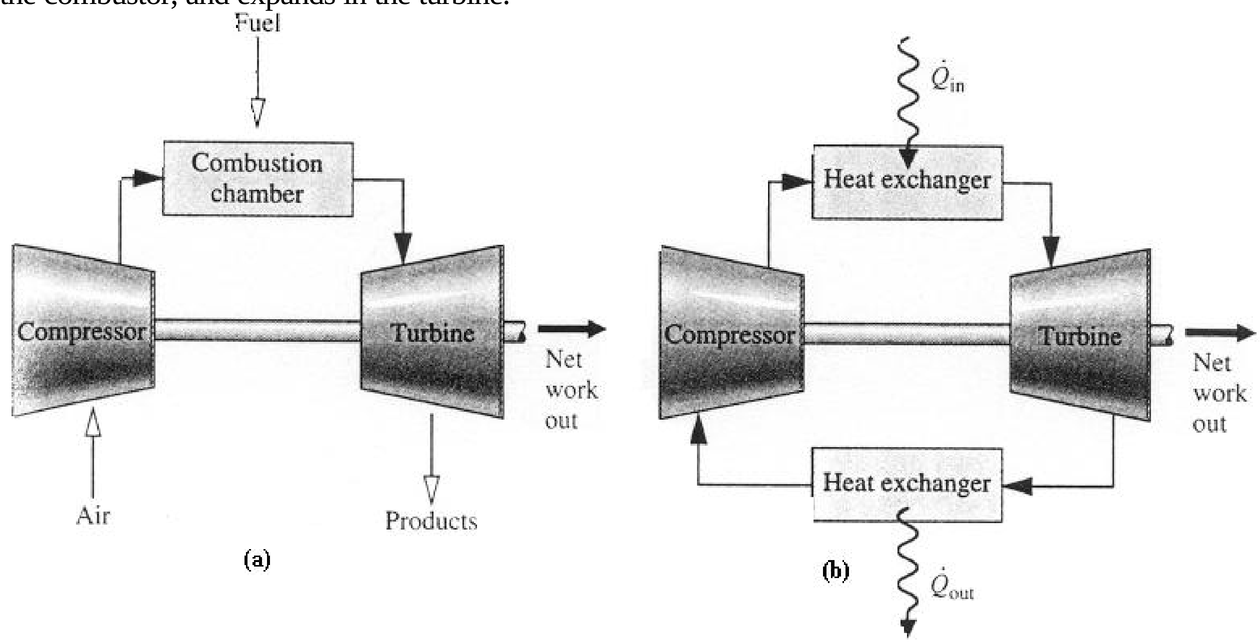
- Utilize gas as the working fluid. During combustion, mixture of (air + fuel) → combustion products. They are lighter and more compact than the vapor power plants examined earlier.

- In the power industry, this all-gas cycle is named combustion turbine (CT). It usually comes as a complete package ready to be put to use and generate power.

- Let's begin this module with a display of some of the web sites which deal with the gas turbine power cycle. Try the following sites:

http://ares.ame.arizona.edu/publications/ices94-paper.pdf – A research report detailing the design and use of a reverse Brayton Cycle heat pump.
http://starfire.ne.uiuc.edu/ne201/course/topics/energy_cycles/simple_brayton.html – Covering open and closed Brayton cycles.
http://www.geocities.com/dthomas599/Project.htm – A paper detailing the design of an isentropic turbojet.
http://www.mech.utah.edu/~issacson/tutorial/BraytonCycle.html – An overview of Constant and Variable Specific Heat Models as well as Java applet calculators.
http://www.freebyte.com/cad/utility.htm and http://er-online.co.uk/s-analys.htm – Free engineering demonstrators for much more than just Thermodynamic applications.
http://members.aol.com/engware/calc3.htm - A page put together by Engineering Software featuring Constant Specific Heat Model calculators for the Carnot, Brayton, Otto, and Diesel cycles.

The gas turbine power plant can be described by considering air flowing into the compressor, getting heated in the combustor, and producing power by interacting with the turbine blades. Air is considered to be the working fluid where it is compressed in the compressor, receives heat from an external source in the combustor, and expands in the turbine.



Simple gas turbine: (a) Open to the atmosphere (b) Closed [2]

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Problem: Air enters the compressor of a gas turbine power cycle at 100 kPa, 300 K, with a volumetric flow rate of 5 m^3/s. The compressor pressure ratio is 10. The turbine inlet temperature was measured to be 1400K. Determine:

a). η- thermal efficiency

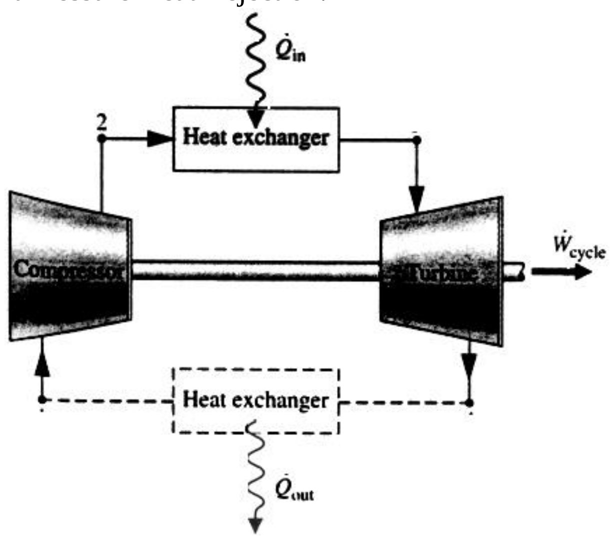
b). $\dfrac{\dot{W}_C}{\dot{W}_T}$ = back work ratio

c). The net power developed, in kW.

Modeling: To solve this problem, we model what is happening with an air-standard Brayton cycle. The Brayton cycle is the ideal cycle for gas turbines. Assumptions for Brayton cycle analysis: (1) There are *four internally reversible processes*, (2) The working fluid is air, (3) Heat is added to the air somehow (simulating the process in the burner), and (4) the cycle is complete by having a heat exchanger between the turbine exhaust and the compressor intake.

Å ***The Brayton Cycle:*** The air- standard Brayton cycle is the ideal cycle for gas turbines.

All 4 processes are internally reversible:
(1) → (2): Isentropic Compression
(2) → (3): Constant Pressure Heat Addition
(3) → (4): Isentropic Expansion
(4) → (1): Constant Pressure Heat Rejection.



Air-standard gas turbine cycle [2]

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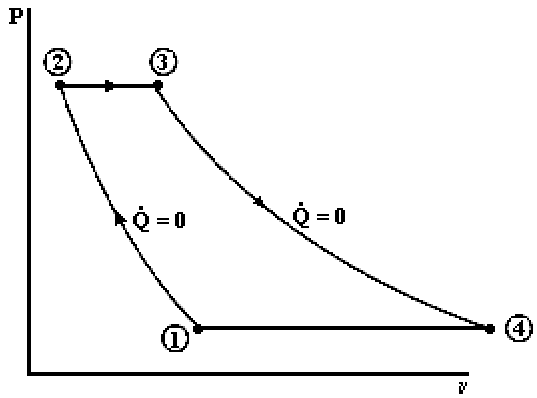
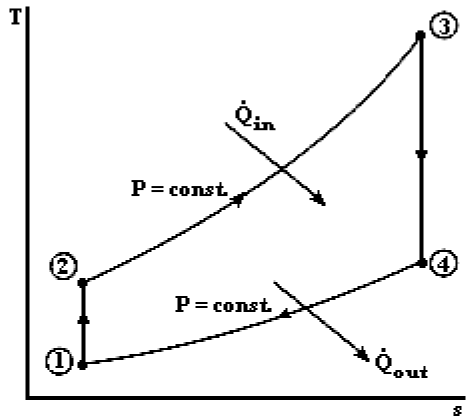
Å ***The Brayton Cycle (continued):***





The thermal efficiency of the Brayton cycle is given by:

$$h_{th} = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{\dot{Q}_H - \left|\dot{Q}_L\right|}{\dot{Q}_H} = 1 - \frac{\left|\dot{Q}_L\right|}{\dot{Q}_H}$$

$$h_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{\dot{W}_T - \left|\dot{W}_C\right|}{\dot{Q}_H} = \frac{(h_3 - h_4) - \left|h_1 - h_2\right|}{h_3 - h_2} \qquad \text{(I)}$$

Thus, the Variable Specific Heat Model dictates finding all four enthalpy values. In order to analyze the processes, we use relationships such as:

$$\frac{p_3}{p_4} = \frac{p_2}{p_1}; \text{ and}$$

(1) → (2) is isentropic: $p_2 / p_1 = p_{r2} / p_{r1}$

(3) → (4) is isentropic: $p_3 / p_4 = p_{r3} / p_{r4}$

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Å **_The Brayton Cycle (continued):_**

- **<u>_Special Case:_</u>** *Cold-Air Standard Analysis* assuming the specific heats to be constant and evaluated at a base temperature of 300 K or 520 °R.

When the specific heats are assumed to be constant: $\Delta h = c_p \Delta T$

In order to analyze the processes, we use relationships such as:

$\frac{p_3}{p_4} = \frac{p_2}{p_1}$; and

(1) → (2) is isentropic: $\frac{p_2}{p_1} = (\frac{T_2}{T_1})^{\frac{k}{k-1}}$;

(3) → (4) is isentropic: $\frac{p_3}{p_4} = (\frac{T_3}{T_4})^{\frac{k}{k-1}}$;

(I) becomes $h_{th} = 1 - \frac{T_1}{T_2} = 1 - \frac{1}{(\frac{p_2}{p_1})^{\frac{k-1}{k}}}$ (II)

i.e. η_{th} is a function of pressure ratio if constant specific heats are assumed.

Note: Actual cycles differ from ideal cycles by having irreversibilities and pressure drops.

- Each device (turbine, compressor, heat exchangers) is an open system. In this analysis, we normally <u>assume</u>:
  - $\Delta KE = \Delta PE = 0$
  - No heat loss between devices
  - SSSF through devices
  - One-inlet, one-exit and 1-Dimensional flow

The first law of thermodynamics for an open system is a statement of the Energy Balance:

$$\frac{dE_{C.V.}}{dt} = \dot{Q}_{C.V.} - \dot{W}_{C.V.} + \sum_i \dot{m}_i (h_i + \frac{V_i^2}{2} + gz_i) - \sum_e \dot{m}_e (h_e + \frac{V_e^2}{2} + gz_e)$$

Under <u>SSSF, Negligible ΔKE and ΔPE, Single-Inlet, Single-Outlet</u> conditions:

Combustor (HX1):	$\dot{Q}_{In} = \dot{m}(h_3 - h_2)$	or	$q_{In} = h_3 - h_2$	∴	$p_2 = p_3$;	Heat In
HX2:	$\dot{Q}_{Out} = \dot{m}(h_1 - h_4)$	or	$q_{Out} = h_1 - h_4$	∴	$p_1 = p_4$;	Heat Out
Turbine:	$\dot{W}_T = \dot{m}(h_3 - h_4)$	or	$w_T = h_3 - h_4$	∴	$s_4 = s_3$;	Work Out
Compressor:	$\dot{W}_C = \dot{m}(h_1 - h_2)$	or	$w_C = h_1 - h_2$	∴	$s_1 = s_2$;	Work In

$$h_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{\dot{W}_T - \left|\dot{W}_C\right|}{\dot{Q}_H} = \frac{(h_3 - h_4) - \left|h_1 - h_2\right|}{h_3 - h_2}$$

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- We need the enthalpy values at each state.

$$T_1 \xrightarrow{\text{Table A-22}} h_1 = 300.19\,\tfrac{kJ}{kg},\ p_{r1} = 1.386$$

$$\frac{P_2}{P_1} = \frac{p_{r2}}{p_{r1}} \Rightarrow p_{r2} = p_{r1} * 10 = 13.86 \xrightarrow{\text{Table A-22}} h_2 = 579.87\,\tfrac{kJ}{kg}, T_2 = 574.1K$$

$$T_3 = 1400K \xrightarrow{\text{Table A-22}} h_3 = 1515.42\,\tfrac{kJ}{kg},\ p_{r3} = 450.5$$

$$\frac{P_4}{P_3} = \frac{p_{r4}}{p_{r3}} \Rightarrow p_{r4} = \frac{p_{r3}}{10} = 45.05 \xrightarrow{\text{Table A-22}} h_4 = 808.5\,\tfrac{kJ}{kg}, T_4 = 787.7K$$

- Now we can calculate the $\dot{W}$s and the $\dot{Q}$s.

$$\frac{\dot{W}_T}{\dot{m}} = h_3 - h_4 = 706.92\,\tfrac{kJ}{kg};\quad \frac{\dot{W}_C}{\dot{m}} = h_1 - h_2 = -279.68\,\tfrac{kJ}{kg}$$

$$\frac{\dot{Q}_{In}}{\dot{m}} = h_3 - h_2 = 935.55\,\tfrac{kJ}{kg};\quad \frac{\dot{Q}_{Out}}{\dot{m}} = h_1 - h_4 = -508.31\,\tfrac{kJ}{kg}$$

- Checking for 1st Law satisfaction:

$$\left.\begin{aligned} \frac{\dot{W}_{Net}}{\dot{m}} &= 706.92\,\tfrac{kJ}{kg} - 279.68\,\tfrac{kJ}{kg} = 427.24\,\tfrac{kJ}{kg} \\ \frac{\dot{Q}_{Net}}{\dot{m}} &= 935.55\,\tfrac{kJ}{kg} - 508.31\,\tfrac{kJ}{kg} = 427.24\,\tfrac{kJ}{kg} \end{aligned}\right\} \quad \frac{\dot{W}_{Net}}{\dot{m}} = \frac{\dot{Q}_{Net}}{\dot{m}} \ \text{v}$$

- The Back Work Ratio = $\dfrac{\dot{W}_C}{\dot{W}_T} = \dfrac{|h_1 - h_2|}{h_3 - h_4} = \dfrac{279.68}{706.92} = 39.6\%$

- Determine the Net Power developed for this cycle:

$$\dot{W}_{Net} = \dot{m}\left(\frac{\dot{W}_T}{\dot{m}} - \left|\frac{\dot{W}_C}{\dot{m}}\right|\right)$$

$$\dot{m} = r\dot{\forall} = \frac{\dot{\forall}_1}{v_1}, v_1 = \frac{RT_1}{P_1} = \frac{\left(0.287\,\tfrac{kJ}{kg*K}\right)(300K)}{\left(100\,\tfrac{kN}{m^2}\right)} = 0.861\,\tfrac{m^3}{kg}$$

$$\dot{m} = \frac{5\,\tfrac{m^3}{s}}{0.861\,\tfrac{m^3}{kg}} = 5.81\,\tfrac{kg}{s} \Rightarrow \dot{W}_{Net} = 5.81\,\tfrac{kg}{s} * \left(427.24\,\tfrac{kJ}{kg}\right) = 2481\,\tfrac{kJ}{s} = 2.48MW$$

The following table summarizes the solution:

T_1 K	**T_2** K	**T_3** K	**T_4** K	**P_1** kPa	**P_2** kPa	**P_3** kPa	**P_4** kPa
300	574.1	1400	787.7	100	1000	1000	100
W_C/m kJ/kg	**W_T/m** kJ/kg	**Q_{In}/m** kJ/kg	**bwr** %	**W_{net}/m** kJ/kg	h %	**Net Power** MW	
-279.68	706.92	935.55	39.6	427.24	45.67	2.48	

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- Note that compression of a gas requires quite of bit of energy as the back work ratio for this example is 39.6% and is typically between 40% and 80%. This is a significant back work ratio especially when compared to a steam power plant having a representative bwr value of say 1%.

Simplified Analysis: Cold Air Standard Analysis - Let's solve the same problem using the Constant Specific Heats Model (CSHM), that is using a cold-air-standard analysis with c_p evaluated at 300 K. (c_p = 1.005 kJ/kg K, and k = 1.4).

Using (II), $h_{th} = 1 - \frac{1}{\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}} = 1 - \frac{1}{(10)^{\frac{0.4}{1.4}}} = 48.2\%$ as compared to 45.67% using the Variable Specific Heat Model.

- Let's validate it by using (I), replacing $\Delta h = c_p \Delta T$, and finding the temperatures.

$$T_1 = 300K\,(Given)$$

$$T_3 = 1400K\,(Given)$$

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 300K(10)^{\frac{0.4}{1.4}} = 579.2K$$

$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} = 1400K(.1)^{\frac{0.4}{1.4}} = 725.1K$$

$$h_{th} = 1 - \frac{(725.1 - 300)}{(1400 - 579.2)} = 48.2\%$$ It checks with the calculated value above.

$$bwr = \frac{h_2 - h_1}{h_3 - h_4} = \frac{T_2 - T_1}{T_3 - T_4} = 41.4\%$$ (using constant specifc heats)

$$\frac{\dot{W}_T}{\dot{m}} = h_3 - h_4 = c_P(T_3 - T_4) = \left(1.005\tfrac{kJ}{kg*K}\right)(1400 - 725.1)K = 678.27\tfrac{kJ}{kg}$$

$$\frac{\dot{W}_C}{\dot{m}} = h_1 - h_2 = c_P(T_1 - T_2) = \left(1.005\tfrac{kJ}{kg*K}\right)(300 - 579.2)K = -280.6\tfrac{kJ}{kg}$$

$$\frac{\dot{Q}_{In}}{\dot{m}} = h_3 - h_2 = c_P(T_3 - T_2) = \left(1.005\tfrac{kJ}{kg*K}\right)(1400 - 579.2)K = 824.9\tfrac{kJ}{kg}$$

$$\frac{\dot{Q}_{Out}}{\dot{m}} = h_1 - h_4 = c_P(T_1 - T_4) = \left(1.005\tfrac{kJ}{kg*K}\right)(300 - 725.1)K = -427.23\tfrac{kJ}{kg}$$

$$\frac{\dot{W}_{Net}}{\dot{m}} = \frac{\dot{W}_T}{\dot{m}} - \left|\frac{\dot{W}_C}{\dot{m}}\right| = 397.7\tfrac{kJ}{kg}$$

T_1 K	**T_2** K	**T_3** K	**T_4** K	**P_1** kPa	**P_2** kPa	**P_3** kPa	**P_4** kPa
300	579.2	1400	725.1	100	1000	1000	100
W_C/m kJ/kg	**W_T/m** kJ/kg	**Q_{In}/m** kJ/kg	**bwr** %	**W_{net}/m** kJ/kg	h %	**Net Power** MW	
-280.6	678.27	824.9	41.4	397.7	48.2	2.308	

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- **Irreversibilities Effect in the Compressor and Turbine:**

Of course, the gas flowing through the compressor and the turbine does not undergo isentropic processes. Typical isentropic efficiencies for the turbine are on the order of 90%, while that for an axial compressor between 70 and 85%. Because of irreversibilities, the cycle's capacity (Net Power) and thermal efficiency will be reduced.

- Let's redo the previous example allowing the turbine to have an isentropic efficiency of 0.85 and the compressor to have an isentropic efficiency of 0.75.

- The 1st Law analysis has not changed and therefore we need the enthalpy values at each state.

$$T_1 \xrightarrow{\text{Table A-22}} h_1 = 300.19\,\tfrac{kJ}{kg},\ p_{r1} = 1.386\,(Unchanged)$$

$$\frac{P_2}{P_1} = \frac{p_{r2}}{p_{r1}} \Rightarrow p_{r2} = p_{r1} * 10 = 13.86 \xrightarrow{\text{Table A-22}} h_{2s} = 579.87\,\tfrac{kJ}{kg}$$

$$\eta_C = \frac{\dot{W}_s}{\dot{W}_a} = \frac{h_1 - h_{2s}}{h_1 - h_{2a}} \Rightarrow h_{2a} = h_1 - \left(\frac{h_1 - h_{2s}}{\eta_C}\right) = 673.1\,\tfrac{kJ}{kg},\ T_2 = 662.46K$$

$$T_3 = 1400K \xrightarrow{\text{Table A-22}} h_3 = 1515.42\,\tfrac{kJ}{kg},\ p_{r3} = 450.5\,(Unchanged)$$

$$\frac{P_4}{P_3} = \frac{p_{r4}}{p_{r3}} \Rightarrow p_{r4} = p_{r3}/10 = 45.05 \xrightarrow{\text{Table A-22}} h_{4s} = 808.5\,\tfrac{kJ}{kg}$$

$$\eta_T = \frac{\dot{W}_a}{\dot{W}_s} = \frac{h_3 - h_{4a}}{h_3 - h_{4s}} \Rightarrow h_{4a} = h_3 + \eta_T\left(h_{4s} - h_3\right) = 914.54\,\tfrac{kJ}{kg},\ T_4 = 883.56K$$

- Now we can calculate the $\dot{W}$s and the $\dot{Q}$s.

$$\frac{\dot{W}_T}{\dot{m}} = h_3 - h_4 = 600.88\,\tfrac{kJ}{kg};\quad \frac{\dot{W}_C}{\dot{m}} = h_1 - h_2 = -372.91\,\tfrac{kJ}{kg}$$

$$\frac{\dot{Q}_{In}}{\dot{m}} = h_3 - h_2 = 842.32\,\tfrac{kJ}{kg};\quad \frac{\dot{Q}_{Out}}{\dot{m}} = h_1 - h_4 = -614.35\,\tfrac{kJ}{kg}$$

- Checking for 1st Law satisfaction:

$$\left.\begin{aligned}\frac{\dot{W}_{Net}}{\dot{m}} &= 600.88\,\tfrac{kJ}{kg} - 372.91\,\tfrac{kJ}{kg} = 227.97\,\tfrac{kJ}{kg}\\ \frac{\dot{Q}_{Net}}{\dot{m}} &= 842.32\,\tfrac{kJ}{kg} - 614.35\,\tfrac{kJ}{kg} = 227.97\,\tfrac{kJ}{kg}\end{aligned}\right\}\quad \frac{\dot{W}_{Net}}{\dot{m}} = \frac{\dot{Q}_{Net}}{\dot{m}}\ \checkmark$$

- The Back Work Ratio = $\frac{\dot{W}_C}{\dot{W}_T} = \frac{|h_1 - h_2|}{h_3 - h_4} = \frac{372.91}{600.88} = 62.0\%$

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Now, we determine the Net Power developed in the system.

$$\dot{W}_{Net} = \dot{m}\left(\frac{\dot{W}_T}{\dot{m}} - \left|\frac{\dot{W}_C}{\dot{m}}\right|\right)$$

$$\dot{m} = r\dot{\forall} = \frac{\dot{\forall}_1}{v_1}, v_1 = \frac{RT_1}{P_1} = \frac{\left(0.887 \frac{kJ}{kg*K}\right)(300K)}{\left(100 \frac{kN}{m^2}\right)} = 0.861 \frac{m^3}{kg}$$

$$\dot{m} = \frac{5 \frac{m^3}{s}}{0.861 \frac{m^3}{kg}} = 5.81 \frac{kg}{s} \Rightarrow \dot{W}_{Net} = 5.81 \frac{kg}{s} * \left(227.97 \frac{kJ}{kg}\right) = 1325 \frac{kJ}{s} = 1.32MW$$

The following table summarizes the solution:

T_1 K	**T_2** K	**T_3** K	**T_4** K	**P_1** kPa	**P_2** kPa	**P_3** kPa	**P_4** kPa
300	662.46	1400	883.56	100	1000	1000	100
W_C/m kJ/kg	**W_T/m** kJ/kg	**Q_{In}/m** kJ/kg	**bwr** %	**W_{net}/m** kJ/kg	h %	**Net Power** MW	
-372.91	600.88	842.32	62.0	227.97	27.1	1.32	

- Note that because of irreversibilities in the compressor and in the turbine:
  - The efficiency has dropped substantially from 45.7% to 27.1 %
  - The cycle net capacity has dropped from 427 kJ/kg to 228 kJ/kg.
  - The back work ratio has increased from 39.6 % to 62% while
  - The net power developed has dropped from 2.48 MW to 1.32 MW, a decrease of 47 %.

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