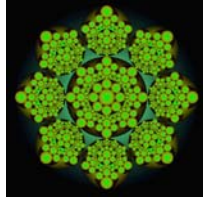


## Kinawa Mathematical Circle



### PROBLEMS - PÓLYA'S METHOD

Four steps:

1. Understand the Problem. Be sure you understand what the problem is asking.
  - ✚ Read carefully, even a few times.
  - ✚ Record the quantities that are given.
  - ✚ Identify the unknowns.
  - ✚ Draw a picture or a diagram.
  - ✚ List all possibilities.
  - ✚ Restate the problem in an easier way if possible.
2. Plan a strategy for solving the problem.
  - ✚ Is there a similar problem you know how to solve?
  - ✚ Can you think of a simpler problem you know how to solve?
  - ✚ Do you need more information?
  - ✚ Why can't you solve the problem, what is missing?
3. Execute your strategy. Change it to a different strategy if necessary.
4. Check and interpret your result. Be sure your answer makes sense.

**Problem 1.** A bathroom scale is set incorrectly not at zero, but otherwise it works fine. It shows 50 lbs when Dan stands on it and 70 lbs when Sarah stands on it, but 115 lbs when Dan and Sarah are both on it. By how much is the scale set too high?



**Problem 2.** In a cross-country run, Jen placed exactly in the middle among all participants, Ann placed lower, in tenths place, and Mary placed sixteenth. How many runners took part in the race?



**Problem 3.** Three prisoners know that the jailer has three white hats and two red hats. The jailer puts a hat on the head of each prisoner and says, "If you can deduce the color of your own hat, you will be freed." One prisoner is blind. The other two can see the hats of the other prisoners but not his own. The first prisoner says "I cannot tell the color of my hat". Then the second prisoner says, "I cannot tell the color of my hat". The third prisoner, who is blind, is able to determine the color of his hat and is freed. What is the color of the blind prisoner's hat?



**Problem 4.** Alice takes one-third of the pennies from a jar. Then Bret takes one-third of the remaining pennies from the jar. Finally, Clara takes one-third of the remaining pennies from the jar, leaving 40 pennies in the jar. How many pennies has Alice, Bret, and Clara taken?

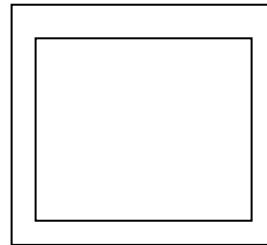


**Problem 5.** You have a strip of paper that is two-thirds of a meter long. However, you need a strip exactly half a meter long. Can you cut off such a length without using a ruler?

**Problem 6.** *Maía bought two unusual sandglasses. One measures a nine-minute interval, and the other measures a thirteen-minute interval. How can she measure a thirty minute-interval using her sandglasses?*



**Problem 7.** A square birthday cake is to be cut into 10 pieces in such a way that all pieces have the same amount of cake and frosting. Assume the perimeter of the cake is 40 inches.





Problem 8. The metropolitan Lansing Area has population of 453,603.

**Population (2009)**

- **City** 113,802
- **Density** 3,403.6/sq mi (1,312.3/km<sup>2</sup>)
- **Urban** 300,032
- **Metro** 453,603

The number of hair strands on a human head varies from person to person, but in general the human head has about **100,000** strands of hair.

The number of strands of hair on a human head varies with color:  
BLONDE - 140,000 strands; BROWN - 110,000 strands; BLACK - 108,000 strands; RED - 90,000 strands.

Why is it that there have to be at least two people living in the Lansing metropolitan area with the same number of hair strands on their heads?

**Problem 9.** There flock of wild gees approached by a domesticated goose, who asked how mane gees were in the flock. She has received the following answer. If you double our flock, add half of our flock and then add a quarter of our flock and at the end add yourself, there would be 100 of us. Now run along and figure out how many of us are in the flock. The goose did, can you?



**Problem 10.** An old fashioned lady has traveled with 10 suitcases. She asked a Bellperson to open his suitcases, gave him 10 keys and left the room. The Bellperson sighed, which key is to which suitcase? "I know that it will always be the last key I try", he thought. Ten attempts for each suitcase, that will be 100 attempts. But perhaps it is not so bad. Can you help him? What is the best way of trying the keys for the suitcases so that the number of attempts be the smallest?



**Problem 11.** A census taker approaches a house and asks the woman who answers the door

*"How many children do you have, and what are their ages?"*

*Woman: "I have three children. If you multiply their ages you get 36, the sum of their ages are equal to the address of the house next door."*

*The census taker walks next door, comes back and says "I need more information."*

*The woman replies "I have to go, my oldest child is sleeping upstairs."*

*Census taker: "Thank you, I now have everything I need."*

**What are the ages of each of the three children?**