NUMB3RS

Supplies: notebook, pencil.

Curriculum (1 hour, 15 minutes)

1. Early man counted by means of matching one group of objects with another group (stones and sheep for example). The operations of addition and subtraction were simply the operations of adding or subtracting groups of objects to the sack or pile of counting stones or pebbles.


A sheep goes out – a pebble is moved from the left pile to the right. When sheep come back, pebbles are moved back. If any pebbles are still in the right pile, there are missing sheep.

2. Each pebble represents one sheep.

Number of pebbles = number of sheep

Each fish has a paper clip attached, each paperclip is attached to a fish, and therefore the number of fish equals the number of paperclips

3. Let's try this with numbers.

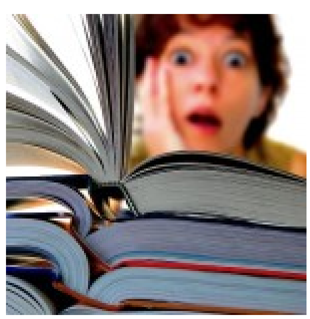
On the left are all numbers: 1, 2, 3, 4, 5, 6, ...

On the right are even numbers: 2, 4, 6, 8, 10, 12, ...

In which basket there is more numbers?

All numbers	—	Even numbers
1	—	2
2	—	4
3	—	6
4	—	8
5	—	10
6	—	12
...	—	...

The objects in both groups are paired, so there is the same number of objects in both baskets!



Surprising, but TRUE!!! Our intuition with infinitely many objects is weak.

4. The numbers we use are Hindu-Arabic, created in India, brought to Europe by Arab traders some 1,000 years ago.

Modern day Arab telephone keypad, with both Western Arabic or European numerals on the left and Eastern Arabic on the right.

5. The Roman numerals were awkward to use.

   I, II, III, IV, V, VI, VII, VIII, IX, X, ..., L, C, M

   For example I + II = III makes sense but IV + VI = X is awkward.

   Look at: XX + XXX = L.

   This system could not be easily used for arithmetic.

Joke: Why nine divided by two equals four?

6. How does our system work?

The number 123 means $1x100 + 2x10 + 3x1$, so

$123 + 567 = 3 + 7 = 10$ units, $2 + 6 = 8$ tens, and $1 + 5 = 6$ hundreds. Since we have an extra one ten from adding units, we have 9 tens together, giving 690 as the answer.

$$\begin{array}{r} 123 \\ +\,567 \\ \hline 690 \end{array}$$

7. A game with a moral:

Choose a number between 1 and 10, triple it, add 6 to the result, triple the result, add the digits, if you have a single digit do nothing, if you have a two-digit number, add the digits again.

Let me guess your result!

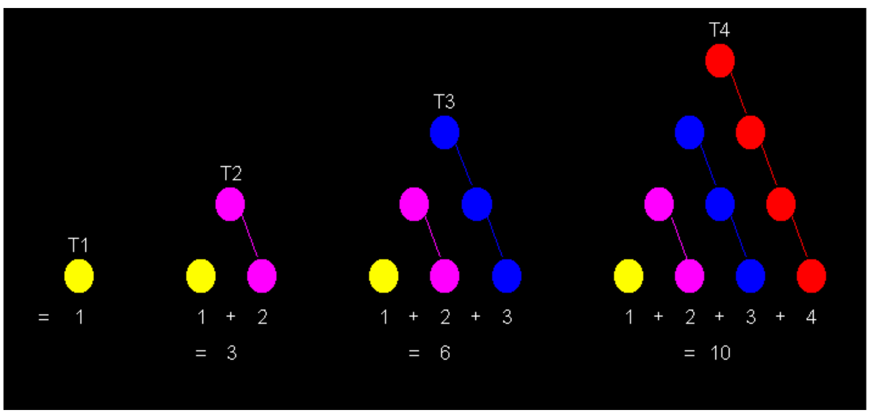
Solution: A – chosen number; $3xA + 6 = 3x(A+2)$;
$3x3x(A+2) = 9x(A+2)$. This number is divisible by 9, and therefore, the sum of its digits is divisible by 9.
Now look at multiples of 9:

Multiple of 9	Sum of digits
9	9
18	9
27	9
36	9
45	9
54	9

63	9	
72	9	
81	9	
90	9	
99	18	Sum of digits = 9
108	9	

8. Activity: add numbers from 1 to 100.
9. Wait! First, do numbers and shapes go together?

    a. Triangular numbers.



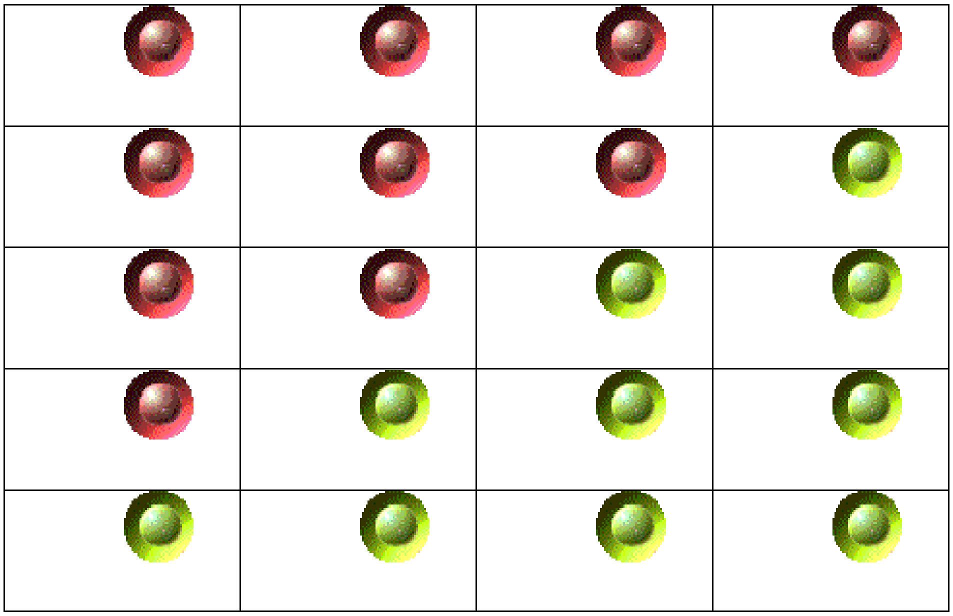
$T_1 = 1$
$T_2 = 1 + 2 = 3$
$T_3 = 1 + 2 + 3 = 6$
$T_4 = 1 + 2 + 3 + 4 = 10$

$T_{100} = 1 + 2 + 3 + 4 + \ldots + 97 + 98 + 99 + 100 = ?$

First look:

$(1 + 100) + (2 + 99) + (3 + 98) + \ldots + (50 + 51) = 50*101$
$= 5050$

Second look:



$2x(1 + 2 + 3 + 4) = 4x5;$

$1 + 2 + 3 + 4 = 4x5/2$

Similarly:

$2x(1 + 2 + 3 + \ldots + 98 + 99 + 100) = 100x101$

$1 + 2 + 3 + \ldots + 98 + 99 + 100 = 100x101/2 = 5050$

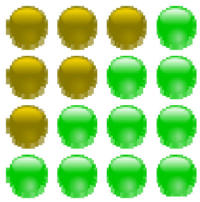
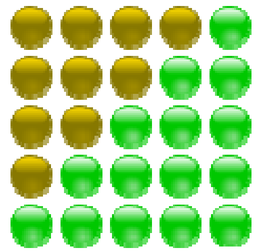
Activity: find the sum: $1 + 2 + \ldots + 48 + 49 + 50 = ?$

b. Square numbers

Look again at the triangular numbers:

$T_1 = 1$
$T_2 = 1 + 2 = 3$
$T_3 = 1 + 2 + 3 = 6$
$T_4 = 1 + 2 + 3 + 4 = 10$
$T_5 = 1 + 2 + 3 + 4 + 5 = 15$
$T_6 = 1 + 2 + 3 + 4 + 5 + 6 = 21$

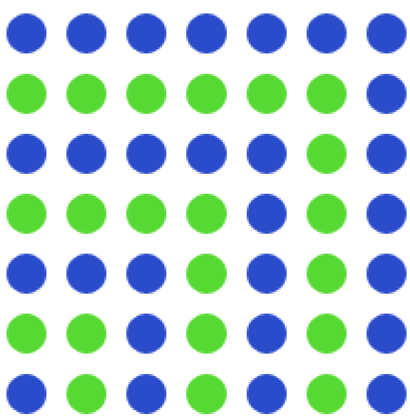
$T_7 = 1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$

6 + 10 = 16  10 + 15 = 25 

$T_3 + T_4 = 6 + 10 = 16 = 4x4 = 4^2$
$T_4 + T_5 = 10 + 15 = 25 = 5x5 = 5^2$

Activity: find the sum of 24th and 25th triangular numbers.

10. Sum of odd numbers.



$1 + 3 + 5 + 7 + 9 + 11 + 13 = 7x7 = 7^2$
We add seven odd numbers

The sum of odd numbers is equal to the square of how many odd numbers are in the sum!

Activity: find $1 + 3 + 5 + \ldots + 95 + 97 + 99$

11. The sum of even numbers.

$2 + 4 + 6 + 8 = 2x(1 + 2 + 3 + 4) = 2xT_4 = 2x10 = 20$

Activity: find $2 + 4 + 6 + \ldots + 96 + 98 + 100$.

12. The Pascal Triangle.

Triangular numbers appear in Pascal's Triangle. In fact 3rd diagonal of Pascal's Triangle, gives all triangular numbers as shown below:

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1

Activity: explain how this happens.

13. Fibonacci Numbers.

Begin with one pair of rabbits (1)

In one month they have a pair of baby rabbits (2)

Month later the first pair has another pair of baby rabbits, the new pair has to mature and has no baby rabbits (3)

Month later the first two pairs have pairs of baby rabbits, the new pair has to mature and has no baby rabbits (5)

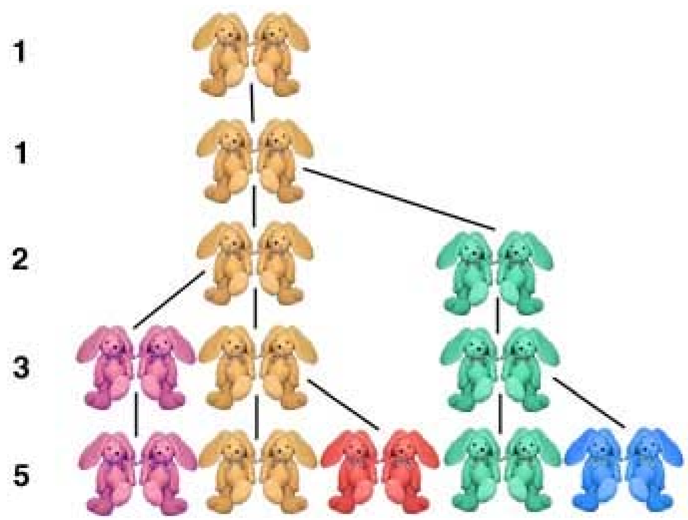
 

Month later each older pair have pairs of baby rabbits, the new pairs have to mature and have no baby rabbits (8)

This process continues, giving Fibonacci numbers:
1, 1, 2, 3, 5, 8, 13, 21, 34, ____, ____, ____,

Another illustration:



Each Fibonacci number is the sum of the two previous Fibonacci numbers, 2 = 1 + 1; 3 = 1 + 2; 5 = 2 + 3;

Activity: write next three Fibonacci numbers:

Activity: Fibonacci numbers in nature.