

In a previous assignment, we saw that the equation

$$e^x = 2x^2 \quad (1)$$

has three solutions on the interval  $-2 \leq x \leq 3$ .

1. Most equations can be expressed in fixed point form ( $x = g(x)$ ) in multiple ways. For example, Eq. (1) can be expressed as

$$x = \pm \sqrt{\frac{e^x}{2}}. \quad (2)$$

2. To approximate the negative root, we must use the negative sign in Eq. (2).

- (a) Plot functions  $x$  and  $-\sqrt{e^x/2}$  on a common plot in Maple on the interval  $-1 \leq x \leq 0$ . Restrict the vertical range to  $-1 \leq y \leq 0$ , make the curves respectively red and blue, give them a thickness of 4 or 5, make the plot size  $300 \times 300$  pixels, and include the plot option `scaling = constrained`.
- (b) Using  $x_0 = -1.0$ , apply 6 iterations of the fixed point to approximate the root near  $x = -0.5$ .

3. To approximate the positive roots, we must use the positive sign in Eq. (2).

- (a) Plot functions  $x$  and  $\sqrt{e^x/2}$  on a common plot in Maple on the interval  $0 \leq x \leq 4$ . Restrict the vertical range to  $0 \leq y \leq 4$ , make the curves respectively red and blue, give them a thickness of 4 or 5, make the plot size  $300 \times 300$  pixels, and include the plot option `scaling = constrained`.
- (b) Using  $x_0 = 1.5$ , apply 10 iterations of the fixed point method to approximate the root near  $x = 1.5$ . What happens? Explain why.
- (c) Using  $x_0 = 2.5$ , apply 10 iterations of the fixed point method to approximate the root near  $x = 2.5$ . What happens? Explain why.

4. Eq. (1) can be expressed in another fixed point form by solving for the  $x$  in the exponent to obtain

$$x = \ln(2x^2). \quad (3)$$

- (a) Plot functions  $x$  and  $\ln(2x^2)$  on a common plot in Maple on the interval  $0 \leq x \leq 4$ . Restrict the vertical range to  $0 \leq y \leq 4$ , make the curves respectively red and blue, give them a thickness of 4 or 5, make the plot size  $300 \times 300$  pixels, and include the plot option `scaling = constrained`.
- (b) Using  $x_0 = 1.5$ , apply 10 iterations of the fixed point method to approximate the root near  $x = 1.5$ . What happens? Explain why.
- (c) Using  $x_0 = 2.5$ , apply 10 iterations of the fixed point method to approximate the root near  $x = 2.5$ . What happens? Explain why.