MATH-305 Proper Integrals: Newton–Cotes Formulas

1. Use the **trapezoidal rule** to approximate $\int_{1.0}^{2.2} f(x)\,dx$ for the following function

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.1	2.2
f	15.4	16.7	18.1	19.7	21.5	23.5	25.8	28.3	31.1	34.2	37.6	41.4	45.7

   (a) using abscissas with spacing $h = 0.1$, Answer: 32.845

   (b) using abscissas with spacing $h = 0.2$. Answer: 32.930

   (c) Apply Richardson extrapolation to obtain an improved estimate. Answer: $32.81\overline{6}$

   (d) For comparison, the **correct** value of the integral is 32.8190 3976 8921.
   Compute the **permille** errors in (a–c). Answers: (a) $-0.791‰$ (b) $-3.381‰$ (c) $0.07231‰$

   (e) Compute the ratio of errors (b)/(a). Does this error ratio make sense for the trapezoidal rule?

2. Repeat problem (1) above using Simpson's–1/3 rule.

   (a) Answer: $32.81\overline{6}$ Error: $0.07231‰$

   (b) Answer: 32.820 Error: $-0.02926‰$

   (c) Answer: $32.816\overline{4}$ Error: $0.07908‰$

   Notice that Simpson's–1/3 rule using **all** data (2a) gave the same answer obtained when using the trapezoidal rule with Richardson extrapolation (1a).

   Why is the error of Richardson extrapolation larger than that of (2a)? It's because the values of f given in the table are rounded to the first decimal place. **Truncation error!**

3. A function f is given in tabular form to 6 significant digits:

x	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5
f	2.34000	2.53177	2.73854	2.92973	3.08105	3.17717	3.21337	3.19532

   Use Simpson's–1/3 rule on interval [3.0,5.0] and Simpson's–3/8 rule on interval [5.0,6.5]. Add the results to approximate $\int_{3.0}^{6.5} f(x)\,dx$.

   Answer: 10.228 853

4. Repeat Problem 3 above using Simpson's–3/8 rule on interval [3.0,4.5] and Simpson's–1/3 rule on interval [4.5,6.5]. Add the results to approximate $\int_{3.0}^{6.5} f(x)\,dx$.

   Answer: 10.228 802

5. What are the sources of error in Problems (3) and (4)?

6.

x	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
f	0.938	1.123	1.345	1.567	1.987	2.212	2.876	2.100	1.532	0.876

   Integrate the above data on the whole interval $[0, 0.9]$ using

   (a) Simpson's–3/8 rule with $h = 0.3$ (ans: 1.7035875)

   (b) Simpson's–3/8 rule with $h = 0.1$ (ans: 1.5598875)

   (c) Richardson extrapolation (ans: 1.5580913)